

Chapter 28

Relative Permeability

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Introduction

This chapter was written as an overview of relative permeability: the basic ideas are given and their evolution is traced. Also presented are some laboratory measurement details and comments on the use of relative permeability information in problem solving. Many unresolved issues still exist, regardless of the fact that the literature is quite rich with the descriptions of what previous workers have thought about this complex subject.

Fluid flow is the major transport process that is involved in the recovery of oil, gas, and associated formation waters from subsurface petroleum reservoirs. As a consequence, process descriptions are needed to understand, to forecast, to manage, and to control production operations. Relative permeability is the concept that is often used as a framework for describing two- and three-phase flow of immiscible fluids through porous sedimentary rocks.

The term *permeability* historically has been adopted as a measure of the porous rocks' ability to conduct fluid. If only one fluid is present in the interstices, this transport coefficient is called the *specific* permeability, but otherwise one must make reference to the *effective* permeability of each of the immiscible fluids in the connected pore space. *Relative* permeability by convention is the ratio of the effective to the specific permeability.

In transport theory one deals with *fluxes*, *forces*, and the coefficients by which these variables are interconnected. As soon as the force-flux relationships are established for particular cases, they can be written in such a way that the various permeability functions will appear explicitly. In other words, just as specific

permeability has the sense of a transport coefficient that appears in Darcy's equation for single-phase flow, effective and corresponding relative permeability functions also can be thought of analogously as important transport coefficients by which multiphase flow processes are best described.

Central to the relative permeability idea is that we are dealing with material response types of parameters that cannot be derived from theory alone. On the contrary, laboratory measurements generally also will have to be made. All measurements are to be made in accordance with how the experimental variables properly are defined and used later.

To simplify the discussion, assume that we are dealing with the steady flow of an incompressible fluid that is moving macroscopically in a horizontal direction. Then,

$$q = \frac{kA\Delta p}{\mu L}, \dots\dots\dots (1)$$

where Δp is the pressure drop that can be measured across a specimen sample of length L and cross-sectional area A , and q is the volumetric rate of discharge of the flowing fluid whose viscosity is μ . Eq. 1 serves to prescribe an experimental measurement methodology by which values for k can be determined unambiguously.

In particular, it is seen that a plot of q vs. Δp should be a straight line whose slope is proportional to the specific permeability, k , as long as certain limiting conditions prevail. These are that (1) the fluid is homogeneous; (2) the temperature is constant; (3) the transport process is free of electrokinetic effects (as occur when certain dilute electrolytes are being moved through electrically nonconducting media), and of film surface flow (as oc-

*Author of the chapter on this topic in the 1962 edition was M.R.J. Wyllie

curs when a gas is moving at such low mean pressure that the frequency of molecular collisions at the interstitial surface boundaries becomes important); and (4) the porous rock is sufficiently rigid and inert to preclude rapid changes in the pore geometry.

Accordingly, in what follows, we adopt the idea that if Darcy's law adequately describes single-phase flow, analogs of it can be postulated just as well as useful descriptions of multiphase flow phenomena.

Historical Background

In the 1930's and 1940's, serious ideas about the meaning and measurement of relative permeability phenomena first appeared. Principal authors were Muskat, Botset, Hassler, Leverett, and perhaps a dozen more whose names punctuate the milestones described in the standard references.^{1,2} These people were connected predominantly in one way or another with the U.S. petroleum industry at a time when quantitative study of recovery of hydrocarbon fluids from subsurface sedimentary environments was in its infancy. In focusing on how reservoir fields of fluid flow would be influenced by the nature and number of the coexistent interstitial fluids, the early workers took advantage of the fact that the existing and well-developed understandings about single-phase flow in porous rocks, when generalized, seemed to provide credible descriptions of multiphase flow situations. When L.A. Richards³ published his classic 1931 paper on the flow of capillary-bound moisture in so-called unsaturated soils, Darcy's pioneering work already had stood the test of three-quarters of a century of scrutiny, use, and amplification by diverse groups of technologists that included groundwater specialists as well as petroleum, chemical, and civil engineers.

Framework Ideas

By analogy to Eq. 1, a set of separate equations can be written to describe multiphase flow phenomena under the restricted conditions that no gravity forces are affecting the steady flow of each of the incompressible immiscible fluids, or

$$q_j = (k_j A)(\Delta p_j) / (\mu_j L) = (k k_{rj} A)(\Delta p_j) / (\mu_j L), \dots (2)$$

where the subscript *j* refers to the *j*th fluid phase (oil, gas, water). Following the usual terminology, *k_j* is called the effective permeability, *k_{rj}* is called the relative permeability, and *p_j* denotes values of pressure locally measured in the various fluid phases that are separated by interfaces of contact.

In particular, *k_j* and *k_{rj}* are to be considered measures of the flow conductivity afforded by the porous rock when saturated with the immiscible fluid phases in some particular way. As will be seen, reference must be made to fluid saturation configurations and distributions as well as to fluid saturation levels. For example, and in analogy to the porosity concept where ϕ is given by the local ratio of pore to bulk volume, the saturation of the *j*th fluid, *S_j*, is defined as

$$S_j = \frac{V_{fj}}{V_{pt}}, \dots (3)$$

where *V_{fj}* is the volume of fluid *j* and *V_{pt}* is the total pore volume. In other words, the product ϕS_j can be thought of as an effective porosity of that portion of the partitioned pore space occupied by the *j*th fluid phase.

It follows that locally (i.e., in any representative volume element of the reservoir rock system of interest) $\sum S_j = 1$, even when saturation is changing with time and position. In any case, it would appear that the relative permeability, *k_{rj}*, has a first-order dependency on the saturation level, *S_j*. But this is only one of the dependencies, because usually many interstitial fluid phase distributions are possible for each level of saturation. Also, the reduced pore space occupied by each of the several pore fluid saturants is not necessarily everywhere bounded by interstitial (solid pore wall) surfaces; hence, it is possible that the influence of prevalent fluid/fluid interfacial boundaries also may have to be considered. Thus, the statement

$$0 < k_{rj} < 1 \text{ for } 0 < S_j < 1, \dots (4)$$

while always true, ignores the fact that *k_{rj}* may equal zero even when *S_j* is finite, and may be greater than unity even for values for *S_j* less than unity.^{4,5} Moreover, Eq. 4 is too crude to indicate explicitly that *k_{rj}* can have more than one value for each value of *S_j* (e.g., because of hysteresis).

Hysteresis

From the thermodynamic point of view, the fluid flow processes under consideration are irreversible (i.e., nonequilibrium); therefore, they are inherently path-dependent. One consequence is that the equilibrium states approached in one direction can be different from those approached in another. This phenomenon is called hysteresis, and explanations for it are not hard to find. For example, while specific permeability depends mostly on the interstitial pore geometry, effective (hence, relative) permeability depends on the fluid saturation geometry as well. Usually there will be more than one way a given fraction of the pore space can be occupied by each fluid phase of interest. The result is that relative permeability data give values that are functions of the history and sequence of the prior saturation changes as well as being merely functions of the fluid saturation levels.

As will be seen later in this chapter, little laboratory work has been published to prove that Eq. 2 truly describes multiphase flow phenomena. Nor can much reassurance be taken from the fact that the validity of Eq. 1 as a description of the characteristics of single-phase flow has been confirmed experimentally for a large number of cases. This is because the unsteady states are of the greatest interest when dealing with petroleum recovery problems, but this aspect is ignored by the Darcian modeling.

In other words, Eq. 2 by itself does not make it possible to account for and to predict the saturation changes that occur as flood fronts move through reservoir space during production. And in the real processes to be modeled, the various phases may be flowing in separate directions rather than colinearly. The greatest limitation of Eq. 2, however, is that it gives no explicit prescription

of how to avoid end effects (the buildup of wetting fluid saturation levels at surfaces of capillary discontinuity such as core end-faces) in laboratory work.

Immiscible Wetting and Nonwetting Pore Fluids

Whenever wetting and nonwetting immiscible fluids compete to occupy the same pore space, it is clear that at inflow and outflow surfaces of so-called capillary discontinuity, there will tend to be a buildup of wetting fluid saturation levels during the course of multiphase flow processes. This has been understood from the earliest days.^{3,6} The central idea is that immiscible fluids that are co-existent in contiguous capillary pore space generally will be separated by curved interfaces of contact instead of by stress-free flat interfaces at contacts that occur exterior to the flow system.

In fact, the interfacial curved boundaries are a reflection of the balance between capillary and gravity forces in the static (stationary) cases, and of the viscous forces as well in the dynamic cases. This means that locally there usually will be a pressure difference between the immiscible fluids. This pressure difference, commonly called the capillary pressure, P_c , by convention is defined as the local difference between p_n and p_w (where the subscripts n and w refer to the nonwetting and the wetting fluids, respectively).

In systems at equilibrium, immiscible fluids tend to be distributed such that the free surface energy of the system is at a minimum, subject to the constraints imposed by the S_j levels and by the hysteretic path of saturation change being followed as successive equilibrium states are being established. This usually means that the wetting fluid will be found in the smaller pore spaces, and that the interfaces of contact will be concave toward the nonwetting fluid (hence, P_c will have positive values). In fact, capillary pressure, like k_j and k_{rj} , frequently is assumed to be primarily saturation-dependent, but it depends substantially on the fluid/fluid interstitial configurations at each saturation level.

The Buckley-Leverett equation⁶ takes the following special form in describing two-phase displacement processes involving one-dimensional (horizontal) flow of incompressible fluids. With density $\rho_j = \text{constant}$ and $\partial q_j / \partial x = 0$, we have

$$\begin{aligned} &\phi(\partial S_w / \partial t) + \partial \{ [(q_w + q_n)M_w] / [(M_w + M_n)A_f] \} / \partial L_h \\ &+ \partial [(M_n M_w)(dP_c / dS_w)(\partial S_w / \partial L_h) / (M_w + M_n)] / \partial L_h \\ &= 0. \dots\dots\dots(5) \end{aligned}$$

where ϕ is the local porosity independent of time, A_f is the cross-sectional area of flow path, P_c is the capillary pressure, L_h is the horizontal distance parameter, and t is the time (both independent variables); and M_n and M_w are the mobilities of the nonwetting and wetting fluids, respectively (i.e., $M_j = k_j / \mu_j$). As can be seen from the derivations of Eq. 5 given in Refs. 1, 2, and 6, the saturation levels vary with position and time in a way that depends on the variation of P_c with S_w , on the saturation gradient, on the relative permeabilities, and on the initial and boundary conditions of the problem [as well as on certain controllable and disposable variables

such as porosity, fluid viscosities, the specific permeability, and the total flow rate given by $(q_w + q_n)$].

Indeed, the importance of having relative permeability information is to make it possible to solve reservoir engineering problems modeled by process descriptions such as given by Eq. 5. The more elaborate modeling technique involving equations with the transport and interaction coefficients described by Rose⁷ require considering more than simple relative permeability information, however. Even so, the experiments prescribed by the Buckley-Leverett⁶ and Rose⁷ equations will not be easy to perform. The laboratory difficulties obviously multiply if unsteady states are to be considered (where S_j changes with time and spatial position). This is why only state-of-the-art methodologies will be described here. Nonetheless, it is clear that future workers will continue to be challenged by the need to develop and to perfect measurement methods that are based on modeling that goes beyond the simplistic Darcian reasoning embedded in Eq. 2.

Measurement Methodologies

A number of measurement methodologies have been described in the literature; these are classified and explained to some extent as follows, along with some of the data that have been reported. Two controversial questions are (1) do the various methods yield equivalent data, and (2) if not, which are the most trustworthy ones?

Laboratory measurement techniques for relative permeability determination are of two sorts. In the so-called steady-state methods, the effective permeability as a function of saturation is calculated from the flow data that are obtained on the assumption that Eq. 2 is correct in form. The trick is to make direct measurements prescribed by the theory, of parameters such as volumetric flow rates, pressure drops, and fluid saturation levels. In one variant of the method, known as the Hassler technique,⁸ provision also is made to control and to measure the local values of capillary pressure to avoid the troublesome end effects.

The so-called unsteady-state methods are based on using integrals of Eq. 5 as the process model. The idea is to observe the consequences (i.e., the outcomes in terms of cumulative production) of controlled multiphase displacement experiments, and then to back-calculate the relative permeability values that are consistent with, and serve to explain, those outcomes. The cumulative production data also are processed to provide a basis for calculating average saturation levels to be associated with the relative permeability values.

As will be seen, the steady-state methods are more time-consuming than the unsteady-state methods; still, the data obtained by them are at least as believable as the plausible model on which they are based (Eq. 2), especially if convincing measures are taken to minimize the capillary end effects mentioned previously. On the other hand, the unsteady-state methods, comparatively speaking, supply the wanted data quickly and cheaply. This latter advantage, it may be argued, is only a partial compensation for the uncertainties in data interpretation inherent in the indirect nature of such measurements. To expose the rationale for selecting one type of procedure over another, the ideas of recent authors are reviewed below.

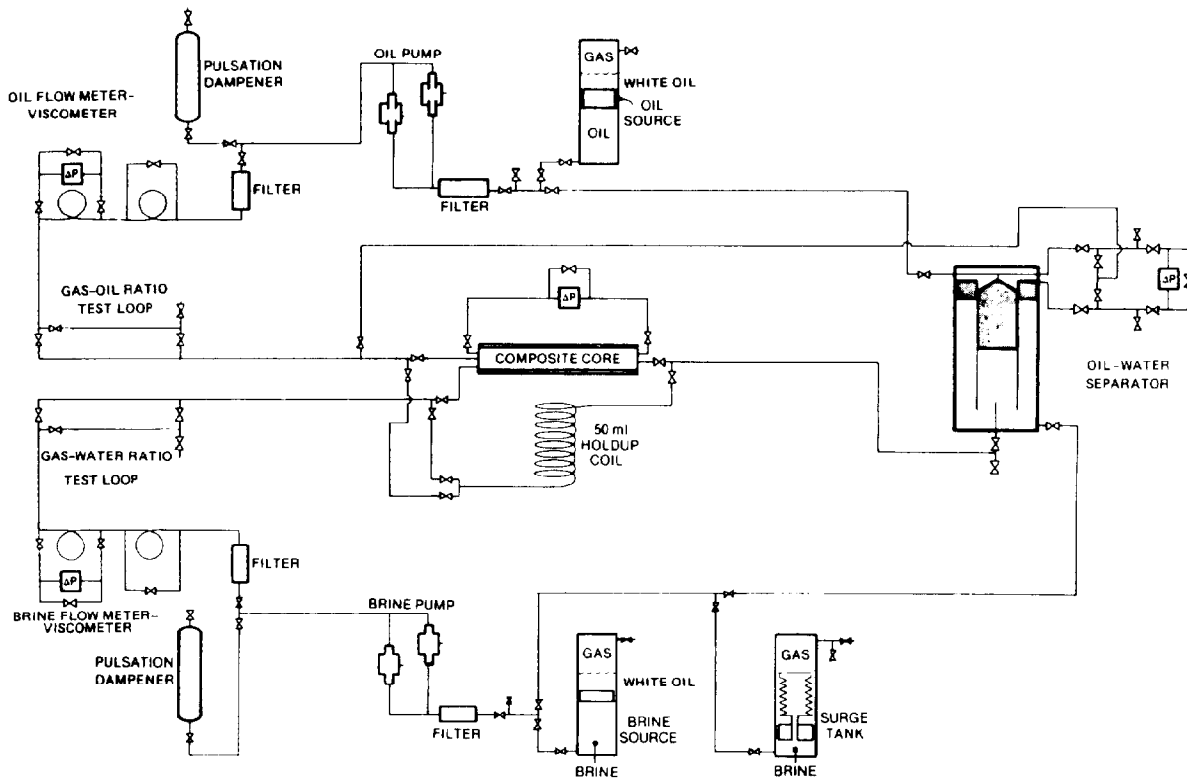


Fig. 28.1—Details of steady-state relative permeability apparatus.

Steady-State k_r Methods

Experimental Procedure. Blackwell and Braun⁹ provide a comprehensive statement of how some people think the steady-state relative permeability method should be practiced when given the understandings and instrumentation opportunities available at the start of the 1980's. Fig. 28.1 is a schematic of the laboratory system that can be used. Positive displacement or other types of constant-rate pumps, one for each fluid, discharge a fixed-ratio mixture into the core sample. Regardless of the initial saturation conditions within the core sample, the effluent fluid mixture eventually will be identical in composition to that being delivered by the pumps upstream. At this steady-state condition, effective permeabilities for the immiscible fluids can be calculated because the separate pump rates (q_o and q_w) will be known, and because an approximation of the pressure drop across the core sample can be used as the indicated driving force for each fluid.

For example, suppose that the reservoir process under consideration involves edgewater encroachment into a uniform sand that has a certain level of interstitial water saturation, with the rest of the pore space filled with an unsaturated oil. A core sample representative of the formation could be selected and, after cleaning, an appropriate initial condition with respect to water and oil saturation could be established by the so-called restored-state capillary pressure technique. Alternatively (and perhaps ideally), a "fresh" core sample could be brought into the laboratory as recovered by a pressure core barrel so that the wanted initial saturation conditions already would prevail. Or, as still another preparation

procedure, an "as-received" core could be processed so that the mud filtrate was replaced by simulated formation water that, thereafter, was displaced down to interstitial levels by flooding the sample with "live" oil. That is, in one way or another, the imbibition water/oil relative permeability data could be obtained where the experiment is started from a proper initial condition in fluid saturations and saturation distributions. And, if reservoir conditions of wettability were to be somehow preserved or restored as preparations for the experimental work are made, and if the ensuing displacement process were to be undertaken under conditions where reservoir-like overburden stress, pore fluid pressure, and/or temperature were simulated, then so much the better. However, the final saturation levels at steady state still would have to be measured. Several methods are available from which this information can be obtained.

Saturation Measurements. What the Braun and Blackwell method⁹ does is have a downstream water/oil separator where (as shown in Fig. 28.1) the bulk of each effluent phase is directed back to the inflow sides of the respective pumps, but where the differential amount between inflow and outflow of each phase is collected in a column and gravimetrically or volumetrically measured. In other schemes, X-ray absorbers or radioactive tracers can be added to one or more of the flowing fluids so that external instrument scans of the core will give the information that can be converted back to saturation levels. Even more simply, the core sample can be removed from the core holder after each steady-state condition is

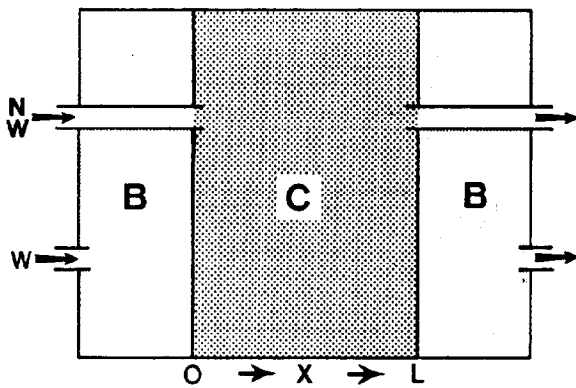


Fig. 28.2—The Hassler "sandwich." C denotes the core sample of length L and B denotes the inflow and outflow capillary end barriers. NW and W designate the ports through which the nonwetting and wetting fluids are directed collinearly as shown by the arrows.

reached, and then weighed so that saturation values can be calculated from independent knowledge of fluid densities and core sample PV.

More details about the steady-state procedure under discussion are available in the references. Suffice it to say that entire curves of relative permeability vs. saturation are to be obtained, following well-defined imbibition or drainage paths, by proceeding stepwise from one steady-state condition to the next. For example, in the water influx experiment under discussion, the q_w/q_o ratio delivered by the pumps could be set at succeeding higher values while going from one steady state to the next until finally the high produced WOR's would confirm that a condition of residual oil saturation had been achieved.

The previous discussion provides a brief description of the sense of the steady-state relative permeability measurement scheme as used when no special effort is made to control end effects (e.g., the buildup of wetting fluid saturation levels at surfaces of capillary discontinuity such as core end-faces). Early data obtained by equivalent steady-state procedures are reported in Muskat's classic work.¹⁰ Remarkably, there does not appear to be much learned from more recently obtained data than was at least qualitatively apparent in the beginning.

For example, one conclusion that can be drawn consistently from laboratory observations is that the inequalities given previously as Eq. 4 can be written more exactly as

$$0 < k_{rj} < 1 \text{ for } S_{ij} < S_j < 1, \dots \dots \dots (6)$$

where S_{ij} is the minimum (irreducible) value of S_j when the j th fluid of interest no longer has phase continuity over sensible distances within the pore space. On the other hand, the idea that relative permeability can be greater than unity for saturations less than unity has been reported^{4,5} and explained as a reflection of the "lubrication" provided as one fluid slides by an adjacent immiscible one.

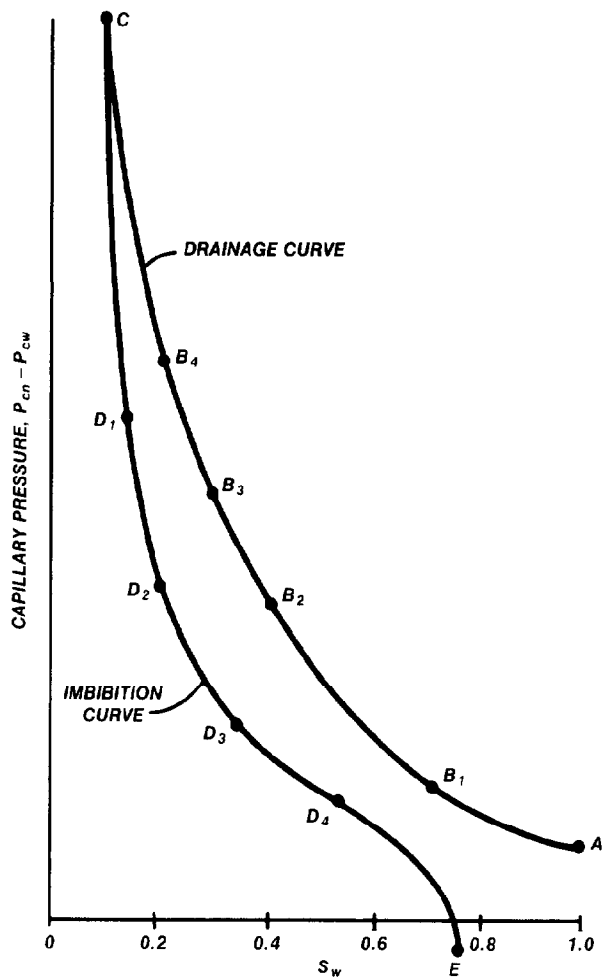


Fig. 28.3—Schematic capillary pressure curves.

On the other hand, when attention needs to be given to end effects, some workers have proposed that long cores be used so that measurements could be confined to an inner portion.¹¹ Even in such a so-called Penn State arrangement, however, the pressure-drop terms of Eq. 2 still are not measured separately for each of the immiscible fluids, and that is why it makes sense to refer back to Hassler's almost-forgotten work.

The Hassler method for relative permeability determination has had a curious history. The patent was granted in 1944⁸ and although cited occasionally, it has been ignored by most workers intent on developing simple state-of-the-art procedures. Scheidegger² referred to the method as superior, but difficult and time-consuming to apply, and at one point Rose¹² gave an analysis of why operational difficulties were to be expected in some applications. More recently, however, a patent has appeared that teaches how Hassler's ideas indeed might be reduced to a practical operating scheme.¹³ Whether or not these claims eventually are substantiated by the facts, it is nonetheless of interest to examine in some detail the principle of the Hassler method for relative permeability determination. Thereby, a reference frame can be established to which other steady-state relative permeability procedures can be compared.

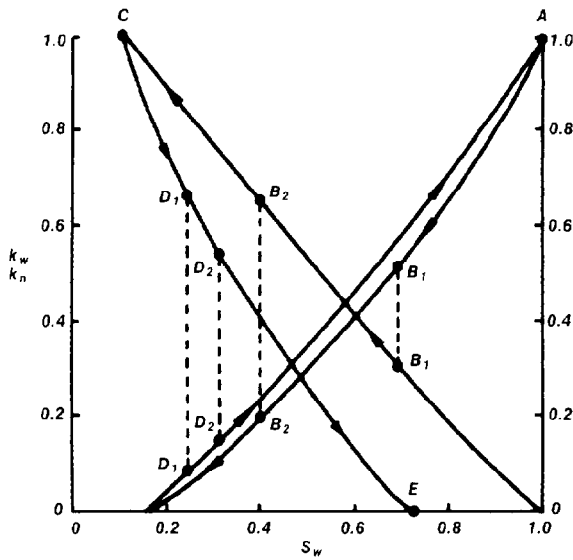


Fig. 28.4—Schematic relative permeability curves.

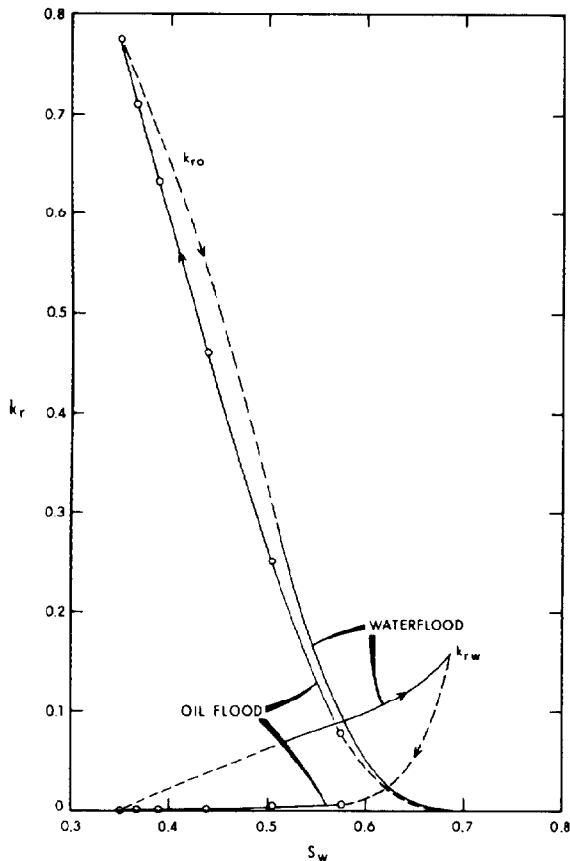


Fig. 28.5—Hysteresis in waterflood vs. oilflood relative permeability curves.

Fig. 28.2 is a schematic that shows the sandwich arrangement where the wetting fluid (denoted by W) passes in series through an inflow capillary barrier as it enters the core sample, and then exits through a similar downstream barrier. The nonwetting fluid (denoted by NW) flows in parallel with the wetting fluid in the core sample but does not enter the pore space of the barriers. This exclusion is achieved by never letting the pressure in the nonwetting phase locally be so great that the barrier threshold pressure is exceeded; hence, the endflow barriers will always remain 100% saturated with the wetting fluid during the course of the test.

Fig. 28.3 is a schematic of a representative capillary pressure drainage and imbibition curve for the reservoir rock sample of interest where the relative permeability relationships are wanted. Suppose that, as an initial condition, the core (as well as the end barriers in series with it) is 100% saturated with water (say with a simulated oilfield brine). Let the threshold pressure for the core sample be P_{cA} and that for the barrier material be greater than P_{cC} . * Threshold pressure is being defined here as the lowest capillary pressure ($P_{cn} - P_{cw}$), where the nonwetting fluid will enter a wetting-fluid-saturated porous medium. Since the capillary end barriers are made of material considerably less permeable than the core sample material, $P_{cC} \gg P_{cA}$.

Hassler's way of performing the relative permeability experiment is to measure the effective permeability to each phase by using Eq. 2 at various successive capillary pressure conditions, such as $P_A, P_{B1}, P_{B2} \dots P_C, P_{D1}, P_{D2} \dots P_E$. The result is a succession of saturation states such as $S_{A1}, S_{B1}, S_{B2} \dots S_{C1}, S_{D1}, S_{D2} \dots S_E$ in the sense that from A to C a drainage curve is being traced since the wetting-phase saturation is always decreasing, while from C to E an imbibition curve is being traced since the wetting-phase saturation is always increasing. Note that at A the wetting-phase saturation, S_A , is 1.0, at C the wetting-phase saturation is close to the interstitial water level, S_{iw} , while at E the wetting-phase saturation is $(1 - S_{rn})$, where S_{rn} designates the residual (nonproducing) nonwetting fluid saturation level.

Corresponding to the saturation points where capillary pressure values have been measured and shown in Fig. 28.3 during drainage and imbibition, the associated relative permeability data to be obtained by the Hassler method are shown in Fig. 28.4 in schematic format. It is instructive to draw some comparisons between these presentations. Capillary pressure as well as relative permeability functions are saturation-dependent. They both appear as parameters in the Buckley-Leverett description of multiphase flow and displacement processes (Eq. 5). Both functions are a direct reflection of the network structure of the pore space (size, shape, orientation, mode of branching, tortuosity, etc.). It is for these reasons that early workers were quick to search for the direct dependency between capillary pressure and relative permeability phenomena¹⁴ and that Hassler, with such great insight, was provoked to emphasize the interconnections that still earlier had been recognized by Leverett¹⁵ and Richards.³

* P_{cC} is the highest value of capillary pressure to be used in the experiment to obtain the lowest value of S_w .

The process under discussion is complicated and not easy to describe without many details. Since these are fully covered in the original references,^{1,2,7} it is enough to end the discussion here with the observation that there are at least two ways, in principle, to practice the Hassler method. One is to impose fixed boundary conditions in pressure upstream as well as downstream, and then to observe flow rates needed for the calculations of permeability data.⁸ The other¹³ is to have constant flow rate as the upstream boundary condition while keeping constant pressure as the downstream boundary condition. What is gained thereby is circumvention of the practical difficulties of avoiding end effects encountered when the boundary conditions are set in terms of upstream and downstream pressure only.

Unsteady-State k_r Method

Compared to the unsteady-state methods to be described now, the steady-state methods are quite straightforward and involve few uncertainties. The following analyses will reveal why this is so.

Experimental Procedure. In so-called unsteady-state procedures, effluent production from a core sample during the course of an imposed displacement process is recorded, and relative permeability functions are generated on the basis of a mathematical modeling of the process that is supposed to be consistent with what is being observed. In practice, the mathematical model usually selected is a simplified form of an integral of the Buckley-Leverett Eq. 5. By linearizing the equation by dropping the capillary pressure (end effect) terms, back-calculating the relative permeability functions appears to be possible.

Since the aim is to get more resolution for calculation of intermediate values of relative permeability and for calculation of the associated saturation values, one needs to spread out the effluent production data. The whole enterprise is compromised, however, when a very *unfavorable mobility ratio* is chosen as the way to prolong the transient period before total breakthrough of the displacing phase. The reader is referred to one analysis of this situation⁷ for further information.

Methods of Calculation

Institut Français du Pétrole Method. The reader also is referred to the definitive paper by authors at the Institut Français du Pétrole¹⁶ for full details about how to practice the unsteady-state method for relative permeability determination. Constant-flow-rate and constant-pressure schemes are described, precautions are enumerated, and calculation schemes are given. Fig. 28.5 is taken from another paper¹⁷ that shows how representative data and the associated relative permeability curves might look in a representative case.

Automated Centrifuge Technique. To finish with the discussion of the variants of the unsteady-state methodologies, it will be useful to cite the work of O'Meara and Lease,¹⁸ who gave three-phase data obtained with an automated centrifuge technique. By spinning core samples in a centrifugal field of known strength and observing effluent volumes as a function of

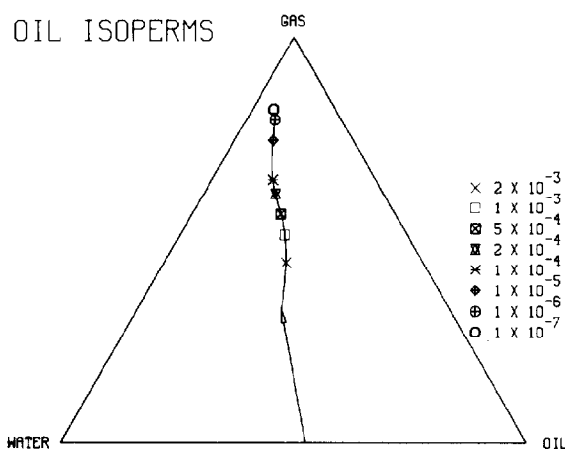


Fig. 28.6—Three-phase saturation trajectory and oil isoperms ranging from 1×10^{-7} to 2×10^{-3} .

time, a back-calculation of relative permeability can be made (which, again, will be at most of qualitative value because of the limiting assumptions that have been used in the mathematical modeling).

Claims made for the centrifuge technique are (1) it does not suffer from viscous fingering distortions, as in the case of the conventional unsteady-state procedures, and (2) it is faster than the competing steady-state procedures. However, the capillary end-effect problem still has to be faced. Another disadvantage is that in any given run, the information obtained applies only to the relative permeability of the invading phase. Since the conventional unsteady-state procedures give relative permeabilities for the displaced (as well as the displacing) fluids, and since larger samples can be processed than is possible in the centrifuge, the conventional approaches are the ones that are considered to be state-of-the-art. In fact, the appeal that the centrifuge technique holds for experimenters is related to its suitability for automation, and to the fact that it is already generating three-phase data. Fig. 28.6, for example, is a display of some of the data that have been reported.

Critique of Methods

To summarize what has been published about relative permeability measurement methodologies, the critique originally given by Scheidegger,² and more recently affirmed,¹⁸ is accepted enthusiastically here. The consensus seems to be that the steady-state methods, Hassler's in particular, give the most believable results since they are based on a plausible (however naive) definition of relative permeability (i.e., as embedded in Eq. 2). Unsteady-state methods, while quicker and easier to apply from the laboratory operational point of view, nonetheless are beset with enormous interpretation difficulties. This is mostly because they involve drawing inferences (by way of back-calculations) from a rough-integral form of the Buckley-Leverett Eq. 5. As noted previously, Eq. 5 is at most an imperfect model of the flow process under study.⁷

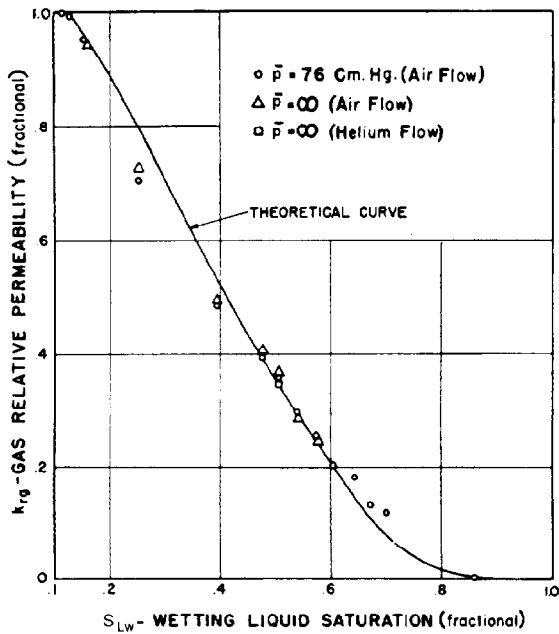


Fig. 28.7—Gas relative permeability vs. total wetting-liquid saturation.

Other k_r Methods

Still other methods to arrive at relative permeability information remain to be cited for completeness. For example, there are the so-called stationary fluid methods¹⁹ and the calculation methods based on capillary pressure and endpoint displacement data associated with Corey *et al.*²⁰ and Stone.²¹ Figs. 28.7 and 28.8 display some of the results given by these investigators.

In the stationary fluid methods, the effective permeability of one phase is measured by flowing that fluid at such a low pressure gradient that the contiguous immiscible fluid is left (it is hoped) unaffected. Clearly, the method can be trusted most when the stationary fluid has a saturation level close to its minimum (irreducible) value (i.e., $S_j \rightarrow S_{ij}$ for the stationary fluid). Otherwise, it may be concluded that the data generated by applying the method will be distorted by the ever-present end effects. On the other hand, the method is easy to apply operationally, and the data generated giving k_o at S_{iw} (irreducible water saturation) and k_w at S_{io} (irreducible oil saturation) are needed along with other parameters when calculated values of relative permeability are to be made by the popular Corey *et al.* and Stone methodologies. In any case, values for k_o at S_{iw} give an indication of the initial productivity of oil wells from horizons where there is only initial oil plus interstitial water saturation. Similarly, values for k_w at S_{io} can be used to indicate the level of the effective water/brine permeabilities when residual oil saturation conditions are reached at the end of a waterflood recovery process.

To understand better the rationale for using relative permeability calculation schemes, reference can be made to the early papers where they were first advocated.^{14,22} All along, the idea underlying them has been that petrophysical properties (e.g., relative permeability and capillary pressure relationships, specific surface area,

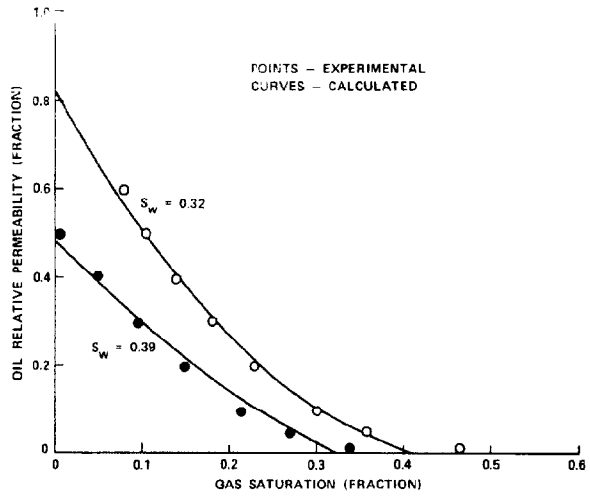


Fig. 28.8—Matching Corey *et al.* Berea sandstone data.

electrical resistivity parameters, etc.) depend in one way or another on the nature of the pore structure, and on how it will be partitioned in representative cases between the intertwining filaments of the adjacent immiscible fluid phases. Of the pore textural properties enumerated previously, relative permeability turns out to be the most difficult one to measure in the laboratory. This has been the origin of the thought (and hope) that an economy could be expected if relative permeability information somehow could be extracted (by way of calculations) from the more easily obtainable data. Wyllie,²³ a foremost advocate of these ways of thinking, is the authority to be consulted when more information is needed.

Eqs. 7a and 7b are the ones given by Corey *et al.*²⁰ for calculating k_{ro} under three-phase saturation conditions, and Eq. 8 is the one proposed by Stone.²¹ For $S_L > S_{Lr}$,

$$k_{ro} = (1 - S_g - S_w)^3 (1 - S_g + S_w - 2S_{Lr}) / (1 - S_{Lr})^4, \dots \dots \dots (7a)$$

and for $S_L \leq S_{Lr}$

$$k_{ro} = (1 - S_g - S_w) / (1 - S_{Lr})^4. \dots \dots \dots (7b)$$

Also,

$$k_{ro} = (k_{row} + k_{rw})(k_{rog} + k_{rg}) - (k_{rw} + k_{rg}), \dots \dots \dots (8)$$

where

- S_L = total liquid saturation, fraction,
- S_{Lr} = residual liquid saturation left in pore space, fraction,
- k_{row} = relative permeability to oil in a gas-free system, and
- k_{rog} = relative permeability to oil in a water-free system.

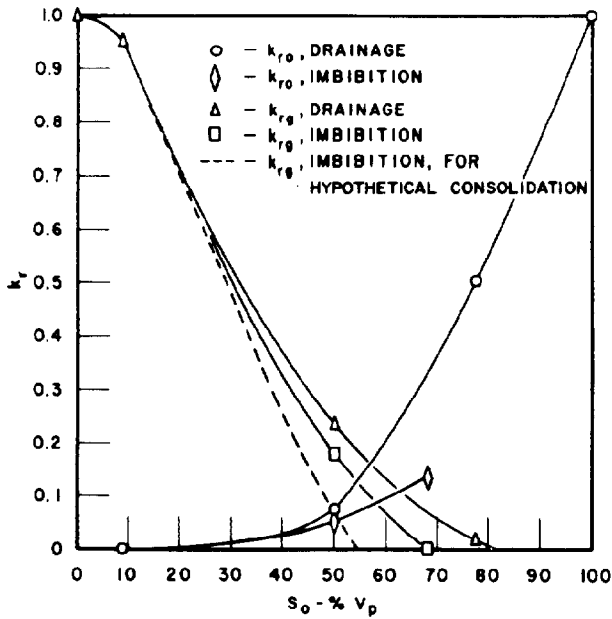


Fig. 28.9—Gas/oil relative permeability in poorly consolidated sandstone.

The recommendation for using Eqs. 7 and 8 to obtain calculated values for the difficult-to-measure k_{ro} under three-phase saturation conditions is that one only needs to enter more easily obtained two-phase data. Inspection will show that while such equations give plausible results (e.g., compatible with the indications of Eq. 6), the results are no more trustworthy than the modeling assumptions when measured relative permeability values are needed but are otherwise unavailable.

Comments. A conclusion to be drawn from this analysis is that if there is any merit to taking short-cuts, it is only when the benefits outweigh the risks. While many (perhaps most) of the readers of this chapter will never have to go to the laboratory to make relative permeability measurements, it is important for them to realize that someone, some day, has to make such measurements before any of the calculation schemes can be used with confidence. In other words, computer guesswork in general will be no substitute for laboratory work. This is because direct observations always will be needed before reliable predictions can be made for processes that inherently depend on material response parameters such as relative permeability.

Recent Literature

More than 30 years have elapsed between an early and a later time when the author of this chapter was provoked to write papers on the then-current problems of relative permeability measurement.^{24,25} It is as though some of the problems have continued to be unsolved and/or unsolvable, while other vexing ones have emerged to take the place of any of the earlier ones that somehow were resolved. In the meantime, work continues on many fronts such as those critiqued in this paper. Note that all the papers now to be referenced have appeared after

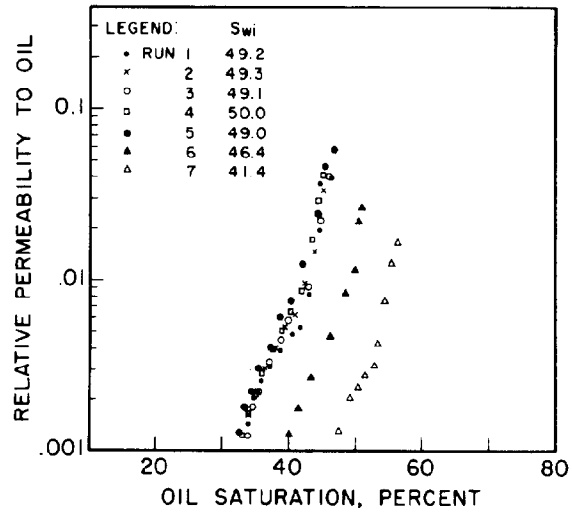


Fig. 28.10—Unsteady-state, three-phase relative mobility (relative permeability to oil, Berea sandstone).

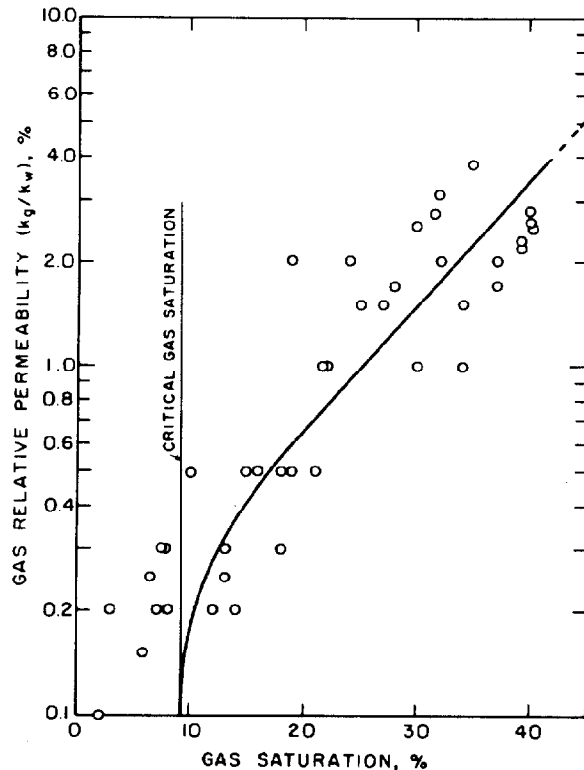


Fig. 28.11—Relative permeability to gas in three-phase flow as a function of gas saturation.

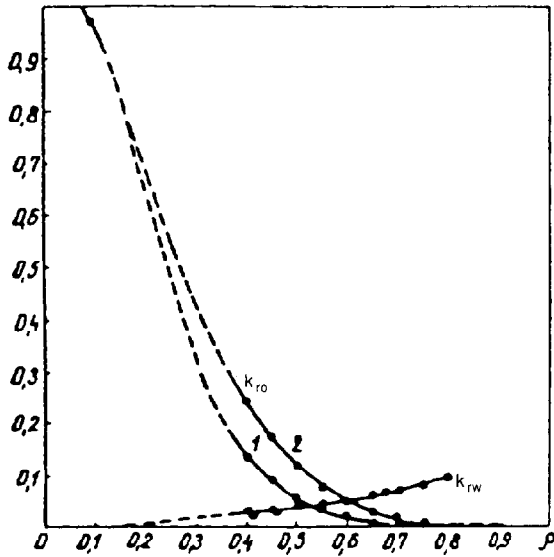


Fig. 28.12—Relative permeability to oil and water showing the effect of displacement (Curve 1) vs. carbonated water displacement (Curve 2).

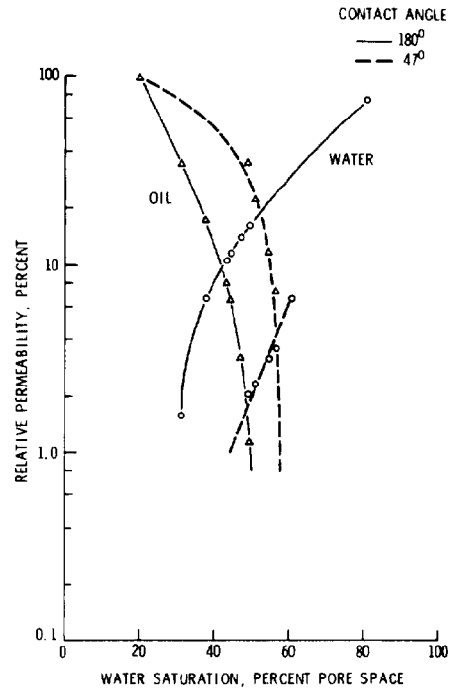


Fig. 28.14—Imbibition relative permeabilities for two wetting conditions, Torpedo sandstone.

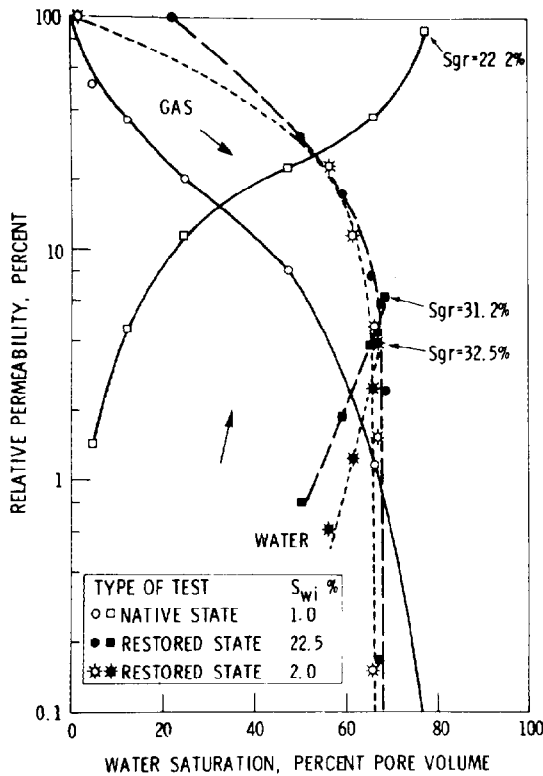


Fig. 28.13—A comparison of restored-state and native-state water/gas relative permeability data.

Wyllie wrote the original chapter on "Relative Permeability" in the first edition of this handbook (in 1962) referencing the literature through the 1950's.²³

Critique of Recent Work

Loomis and Crowell²⁶ present an early but extensive comparison of data obtained by various methods, and as calculated by various mathematical modeling schemes. The high degree of conformity reported makes suspect the objectivity of the work. On the other hand, the authors were quick to point out that wettability was an uncontrolled laboratory variable in their work. This disquieting contention was matched by the surprising result reported without explanation by other workers at the same time, namely that hysteresis effects were different for unconsolidated vs. consolidated core samples.²⁷ Fig. 28.9 shows the trends observed.

The paper of Sarem²⁸ was one of several describing three-phase measurements by an unsteady-state method. One observation (see Fig. 28.10) of this work had to do with the important role played by the initial saturation conditions. At about the same time, Saref and Fatt²⁹ reported success in the development of a nuclear magnetic resonance technique for measurements of fluid saturation levels. These workers, moreover, confirmed the long-held view that relative permeability to gas in three-phase systems depended mostly on the total liquid saturation. This calculation is supported by the data shown in Fig. 28.11.

Two papers appeared in the early 1970's^{30,31} indicating that relative permeability to oil is greatly increased if CO₂ is present (see Fig. 28.12). At this same point in time, Lefebvre du Prey³² was dealing with the question of interfacial tension effects on relative

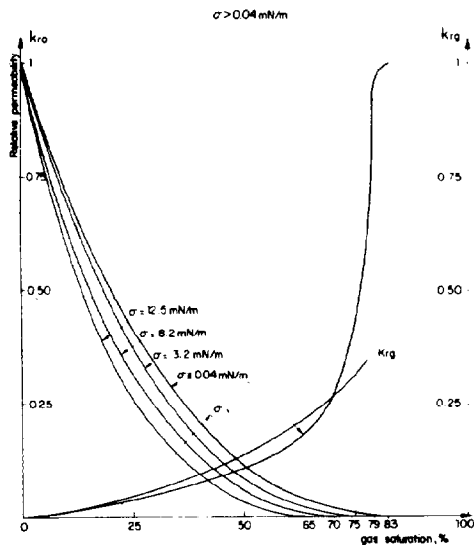


Fig. 28.15—Gas/oil relative permeability calculated from production data from experiments.

permeability while Schneider and Owens³³ and Owens and Archer³⁴ were concluding that because of wettability effects, experimenters were advised to make use of so-called native-state cores. The work of Bardon and Longeron³⁵ also dealt with this subject. And somewhat later Sigmund and MacCaffery³⁶ were trying to sort out the influence of reservoir rock heterogeneity on relative permeability characteristics. Some of the data presented by these authors are shown in Figs. 28.13, 28.14, and 28.15.

In the 1980's, numerous relative permeability papers continue to appear as though a renaissance interest in the old subject is developing. A noteworthy one was by Hagoort,³⁷ who used a centrifuge technique to show the high efficiency of the gravity drainage recovery process in water-wet cores. Bogdanov and Markhasin³⁸ introduced the less familiar subject that speculated that viscosity changes (because of molecular-surface interactions with the rock matrix) could distort relative permeability data. At the same time, Ashford,³⁹ in a very comprehensive paper, was reopening the dubious issue of how relative permeability and capillary pressure data can be linked directly. (As implied above, it is nice to have a conceptual theory available to explain relative permeability effects, but to expect that calculations avoid the need for careful experimental work is a kind of wishful thinking that can be justified only when there is a great urgency to have qualitative inputs for reservoir process simulations). Fig. 28.16 is representative of the fits reported by Ashford between calculated and measured values.

Delshad *et al.*⁴⁰ currently have addressed the interesting question of whether the transport of low-tension micellar fluids will significantly change classical relative permeability trends. They show that the residual (end-point) saturations decrease and relative permeabilities increase as interfacial tension decreases. The predicted nearly 45° relative permeability curves are shown for one case in Fig. 28.17. Yokoyama⁴¹ has been dealing

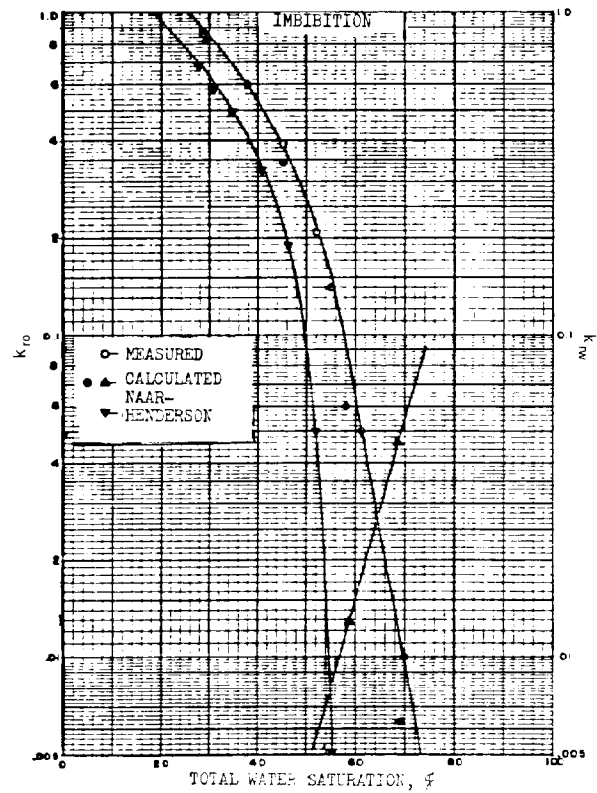


Fig. 28.16—Relative permeability relations for imbibition and drainage experiments, with values calculated by Naar *et al.* method.

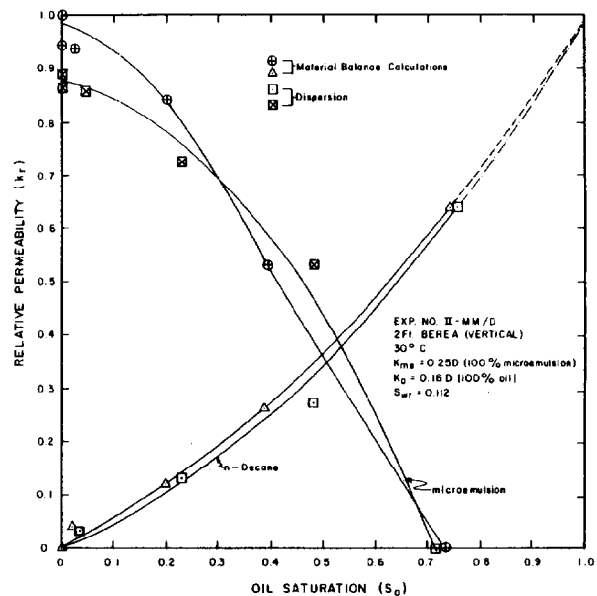


Fig. 28.17—Imbibition relative permeability curves for microemulsion and decane vs. oil saturation in Berea sandstone.

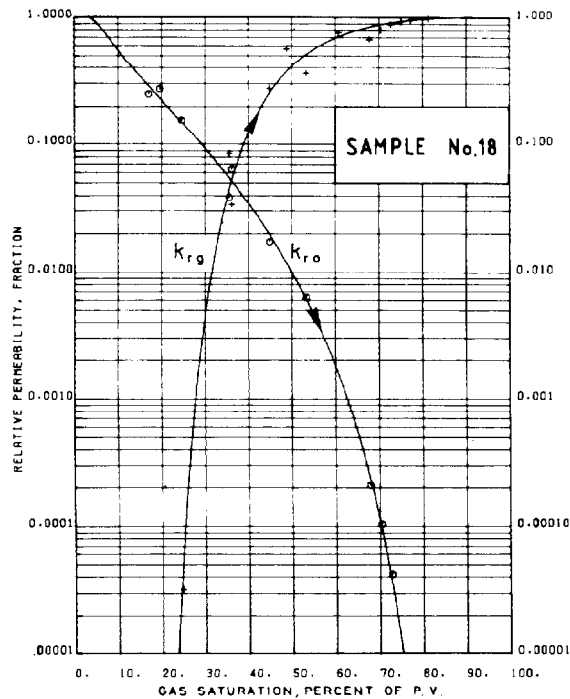


Fig. 28.18—Drainage k_{ro} and k_{rg} curves.

with the equally complex problem of accounting for transverse and longitudinal capillary imbibition during displacements in stratified media.

In another direction, Carlson⁴² extends Land's⁴³ earlier prescriptions for calculating flow from independent measurements of rock properties. Chierici⁴⁴ does the same thing, on the basis of the "bundle of capillary tubes" model described earlier by Brooks and Corey.⁴⁵ Some of these data are shown in Fig. 28.18.

Finally, Lin and Slattery⁴⁶ and Mohanty and Salter⁴⁷ carry pore structure (network) modeling as a basis for arriving at calculated relative permeabilities to still higher plateaus of sophistication. Extensive bibliographies are provided by these latter authors. Some of their results are given in Figs. 28.19 and 28.20.

For example, Fig. 28.20 indicates that (1) relative permeabilities to the nonwetting phase k_{rn} during secondary imbibition (SI) and imbibition (IM) are essentially the same but lower than the values that refer to the primary drainage (PD) conditions, and (2) relative permeabilities to the wetting phase k_{rw} during primary drainage is lower than that in either secondary drainage or imbibition. In other words, those authors⁴⁷ conclude that "the ratio of conductivity or nooks and crannies to that of a full throat feature influences the wetting fluid permeabilities only at low saturation."

Other recent papers are those of Salter and Moharty,⁴⁸ who made observations to justify a modeling of multiphase flow that postulates flowing, dendritic, and isolated configurations for each phase, and of Maini and Batycky,⁴⁹ who claim (see Fig. 28.21) that temperature influences both the endpoint saturation and the shape of the relative permeability curves. A different view about the importance of temperature effects had been expressed earlier by Sufi *et al.*⁵⁰

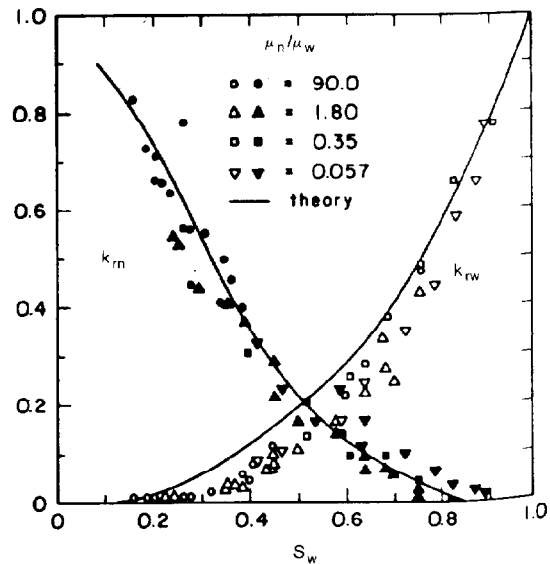


Fig. 28.19—Comparison of drainage relative permeabilities from randomized network model with experimental data for 100 to 200 mesh sand.

The paper of O'Meara and Lease¹⁸ has been cited previously in connection with the unsteady-state determination of three-phase relative permeabilities using the centrifuge technique. It, along with the Maini and Batycky paper,⁴⁹ are just two of nine other papers on relative permeability presented at the 1983 Society of Petroleum Engineers Annual Technical Conference and Exhibition in San Francisco, Oct. 5-8. Listing them by topic is one way to expose the depth and breadth of current interests in the subject. Thus, Kortekass⁵¹ describes displacement in cross-bedded reservoirs. Meads and Bassiouni⁵² speak of combining production history and petrophysical correlations to enhance the representativeness of relative permeability data. Miller and Ramey⁵³ deal further with temperature effects for oil/water systems. Mohanty and Salter⁵⁴ extend their work on oil mobilization, transverse dispersion, and wettability effects. Fulcher *et al.*⁵⁵ and Harbert⁵⁶ deal further with low interfacial systems. Heiba *et al.*⁵⁷ address the wettability questions. And in companion papers, Heaviside *et al.*⁵⁸ cover the experimental and theoretical aspects of relative permeability phenomena. Such are the extensive details currently being discussed.

Ramifications Needing Attention

In summary, the reader who has studied the representative papers cited here, and the even larger number that are scattered in the literature at large, will conclude that this chapter is not the final one to be written on what amounts to be a very complex subject. Some of the ramifications that appear to need further attention include studies of the following effects: (1) phase changes such as gas evolution during multiphase flow, (2) non-collinear flow in a gravity force field between immisci-

ble fluids of differing densities, and counterflow imbibition, (3) reservoir rock anisotropy and heterogeneity by which sample size and sampling frequency requirements are to be determined, (4) "fines" movements, (5) overburden stress simulation and related stress-relaxation and creep compaction phenomena, (6) viscous drag at interstitial interfaces between contiguous immiscible fluids, (7) chemical precipitation and dissolution phenomena, (8) chemical reactions, (9) high Reynolds-number conditions (nonlinear laminar and turbulent flow regimes) (10) Klinkenberg gas-slippage effects, (11) concentration and/or thermal gradients being superimposed on fields of flow primarily caused by mechanical energy gradients, (12) non-Newtonian rheology, (13) systems characterized by more than one local pore space type, (14) fluid/solid interactions, for example, as related to the mineralogy of interstitial clays, (15) viscous fingering, and (16) hysteresis related to wettability changes.

While theoretical considerations may permit a qualitative prediction of the nature of some of these effects, in the final analysis, any truly quantitative assessments should be based on directly undertaken experimental work, if possible. This is not to say that in every case the laboratory data obtained on small hand specimens will reveal everything that needs to be known about large composite petroleum reservoirs, but rather that observations generally are more trustworthy indicators than are blind guesses.

Conclusions

Several questions come to mind whenever the subject of relative permeability is raised: how is it to be defined? where can the information be obtained? why is it needed and by whom? What are the proper (fruitful) ways to use it? Only partial answers to some of these questions have been given in this chapter. This is because the subject is too vast to be dealt with fully in limited space and because the details not covered are probably too specialized for the average reader, whose concern is only with the ordinary applications.

Here we have asserted that definitions of permeability (specific, effective, relative) are embedded in the differential equations that describe the transport equations governing fluid flow in petroleum reservoirs. Eqs. 1 and 5 are examples that apply to special situations (steady single-phase flow and unsteady multiphase flow of immiscible fluids, respectively). Obviously, the differential equations of transport will have other forms for more general cases of fluid flow, such as those observed when there is coupling with chemical diffusion and/or heat transfer processes. This is to say that there may be different kinds of relative permeability to deal with according to the nature of the process under consideration. In all cases, however, one must start by constructing the defining equations to be compatible with the underlying principles of nonequilibrium thermodynamics.

Relative permeability information can be obtained in two major ways. The preferred method is to have an experiment performed on a representative sample of the reservoir rock according to the procedure prescribed by the appropriate integral form of the defining differential equation. Eqs. 1 and 2 are examples of integrals that involve measurable terms (such as volumetric flow rates

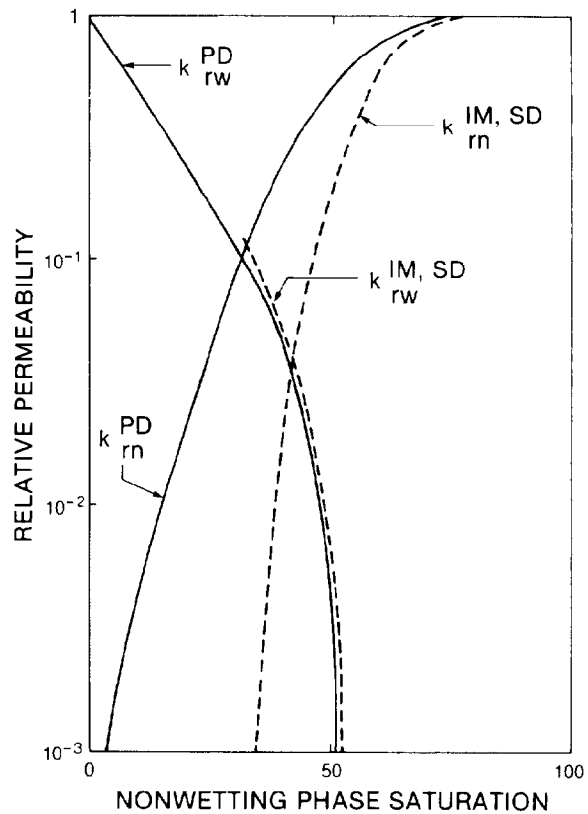


Fig. 28.20—Relative permeability curves.

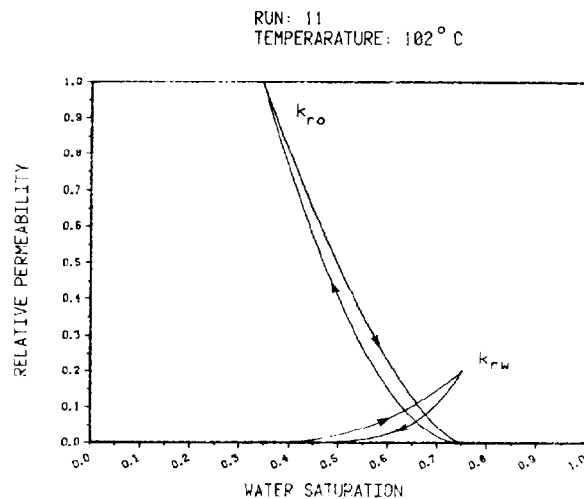


Fig. 28.21—High-temperature relative permeability curves for oil and water.

and pressures, p_i , at bounding sample surfaces). Since the parent differential equations themselves have terms that cannot be directly measured in the laboratory (such as velocity and mechanical energy gradients), integral forms that apply to the particular initial and boundary conditions of the problem must be derived and used for each specific case.

In these connections, it will be revealing (and disturbing) to mention that the degree of equivalence of relative permeability functions obtained by different methods (e.g., the steady- vs. the unsteady-state schemes mentioned previously) so far has not been established fully. Eq. 2 prescribes how the steady-state results are to be obtained, while integrals of Eq. 5 prescribe how the unsteady-state results are to be obtained. Until the definitive laboratory work is done and the comparisons are reported in the literature, users of relative permeability information today will continue to be left in the dark as to which procedure can be used with the most confidence when applications are being made.

As for ordinary applications, relative permeability information is used by petroleum reservoir engineers when interpretations and assessments are being made about the outcomes of observed and probable petroleum recovery processes. These applications are referenced in other chapters in this handbook (e.g., on reservoir simulation, well testing analysis, etc.), and will not be discussed further here. Suffice it to say that the same governing transport equations and their integrals by which relative permeability is defined, necessarily are used again in the ensuing analyses of reservoir performance; hence, relative permeability will be needed as input data whenever analytical studies are undertaken. Among other things, this means that using drainage relative permeability data to describe an imbibition process (such as waterflooding) should be avoided.

A second way to obtain relative permeability information is to develop suitable models of the processes under consideration for use as calculation algorithms. Eqs. 7 and 8 are examples that in use require laboratory data more easily obtained than the relative permeability functions themselves. For example, to calculate three-phase oil relative permeability as a function of oil saturation (where $S_o = 1 - S_w - S_g$) by Eq. 7, all that is needed is prior knowledge of the interstitial water saturation. Naturally, such calculation schemes cannot be used blindly except for cases where it has been independently validated that calculated and experimental values are equivalent. In other words, even the most carefully constructed calculation scheme does not circumvent the need to have experimental measurement methods also developed and available.

However, calculation schemes usually involve analytical functions that can be entered directly into the computer software used for such things as qualitative economic forecasting. Similarly, the related way to obtain relative permeability information, namely by inferring it from the values that force history matches of observed field data, also has a utility when qualitative assessments are being made using reservoir simulators.

The points being made here are both subtle and self-evident. The aim has been simply to convey a certain set of useful ideas to users of relative permeability information such as (1) that relative permeability methodologies

are still in the developmental state even after more than a half century of well-intentioned labor by hundreds of workers and (2) that users of relative permeability information must put the burden of proof on those that supply it, to demonstrate that credible schemes have been used.

Acknowledgment

Any author who has been writing narrowly on a specialized subject like relative permeability for more than a third of a century (specifically from 1948¹⁹ to 1984²⁵) has to be deeply indebted to all those inspiring workers who have managed to keep the issues alive and in focus over such a long period. A few of them, but not all, are named in the abbreviated references that appear here. Likewise, any teacher for a similarly long period must be deeply indebted to the generations of inquisitive students who were willing to ask the provocative questions before they graduated and left to disappear into oblivion. What a lucky thing it was to have had so many companions on what otherwise would have been a dreary Orwellian journey.

Nomenclature

- A = cross-sectional area
- A_f = cross-sectional area of flow path
- k = specific permeability
- k_j = effective permeability, Fluid j (gas, oil, or water)
- k_{rj} = relative permeability, Fluid j (gas, oil, or water)
- k_{rog} = relative permeability to oil in water-free system
- k_{row} = relative permeability to oil in gas-free system
- k_{rw} = relative permeability of wetting fluid
- K_{me} = microemulsion dispersion coefficient
- L = length
- L_h = horizontal distance
- M_n = mobility of nonwetting phase
- M_w = mobility of wetting phase
- p_j = pressure, Fluid j
- Δp = pressure drop
- P_c = capillary pressure
- q = volumetric flow rate
- S_{gr} = residual gas saturation
- S_{ij} = irreducible (minimum) value of S_j
- S_{iw} = irreducible (interstitial) water saturation
- S_j = fractional saturation of j th fluid
- S_L = total liquid saturation
- S_{Lr} = residual liquid saturation
- S_{Lw} = wetting liquid saturation
- S_{rn} = residual (nonproduced) nonwetting fluid saturation
- S_w = water saturation
- S_{wr} = residual water saturation
- t = time
- V_{fj} = volume of Fluid j
- V_{pt} = total PV
- Δp = pressure drop
- μ = fluid viscosity

σ = interfacial tension

ϕ = porosity

Subscripts

g = gas

j = j th fluid (gas, oil, or water)

n = nonwetting

o = oil

w = water or wetting

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