Chapter 34
Wellbore Hydraulics

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Introduction

Wellbore hydraulics is defined here as the branch of production engineering that deals with the motion of fluids (oil, gas, and water) in tubing, casing, or the annulus between tubing and casing. Consideration is given to the relationship among fluid properties, fluid motion, and the well system. More specifically, the material presented is intended to describe methods for solving problems associated with the determination of the relationship among pressure drop, fluid rates, and pipe diameters and length.

To maintain the scope of this section within prescribed limits, some material and data that are pertinent to the solving of wellbore problems, but which can be found conveniently elsewhere, are not presented. The material not covered includes (1) methods of measurement and (2) complete data on fluid properties (See Chaps. 13, 16-19, 24).

The theoretical discussion that follows provides a basis for the development of correlations and calculation procedures in subsequent parts of the section.

Theoretical Basis

Fluids in Motion

Energy Relationships. The energy relationships for a fluid flowing through tubing, casing, or annulus may be obtained by an energy balance. Energy is carried with the flowing fluid and also is transferred from the fluid to the surroundings or from the surroundings to the fluid. Energy carried with the fluid includes (1) internal energy, U, (2) energy of motion or kinetic energy ($mv^2/2g_c$), (3) energy of position (potential energy mgZ/g_c), and (4) pressure energy, pV. Energy transferred between a fluid and its surroundings includes (1) heat absorbed or given up, Q, and (2) work done by the flowing fluid or on the flowing fluid, W.

The conservation of mass, or the first law of thermodynamics, states that the change in internal energy plus kinetic energy plus potential energy plus pressure energy is equal to zero. The following energy balance between points 1 and 2 in Fig. 34.1 and the surroundings illustrates the relationship for the previously listed energy terms for unit mass of fluid.

$$U_2+\frac{v_2^{\,2}}{2g_c}+\frac{g}{g_c}Z_2+p_2V_2=U_1+\frac{v_1^{\,2}}{2g_c}+\frac{g}{g_c}Z_1$$

$$+p_1V_1+Q-W, \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$$

where

U = internal energy,
v = velocity,
g_c = conversion factor of 32.174,
g = acceleration of gravity,
Z = difference in elevation,
p = pressure,
V = specific volume,
Q = heat absorbed by system from surroundings, and
W = work done by the fluid while in flow.

This energy-balance equation is based on a unit mass of fluid flowing and assumes no net accumulation of material or energy between points 1 and 2 in the system.

*Authors of the original chapter on this topic in the 1962 edition included these authors and J.K. Welchon (deceased).

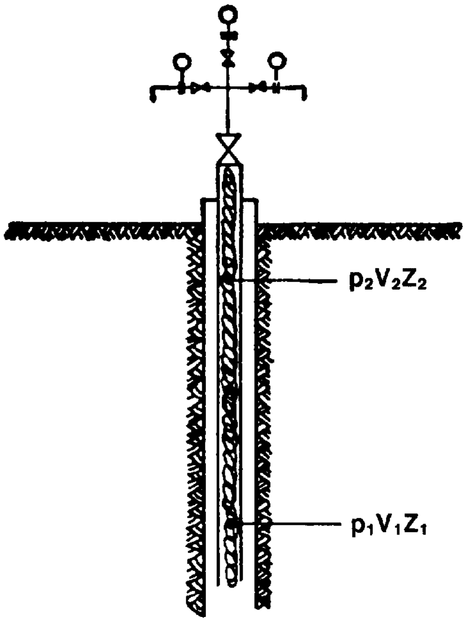

Fig. 34.1—Illustration of energy-balance relationship.

Eq. 1 also can be put in the form

$$\Delta U+\frac{\Delta v^2}{2g_c}+\frac{g}{g_c}\Delta Z+\Delta(pV)=Q-W. \quad (2)$$

since

$$\Delta U=\int_{S_1}^{S_2} T\mathrm{d}S+\int_{V_1}^{V_2} p(-\mathrm{d}V)$$

and

$$\int_{S_1}^{S_2} T\mathrm{d}S=Q+E_\ell$$

where
T = temperature,
S = entropy, and
E_ℓ = irreversible energy losses, and

$$\Delta(pV)=\int_{V_1}^{V_2} p\mathrm{d}V+\int_{p_1}^{p_2} V\mathrm{d}p.$$

Eq. 2 can be put in the more familiar form

$$\int_{p_1}^{p_2} V\mathrm{d}p+\frac{\Delta v^2}{2g_c}+\frac{g}{g_c}\Delta Z=-W-E_\ell. \quad (3)$$

Since, in the system shown in Fig. 34.1, there is no work done by or on the flowing fluid, W is equal to zero and the following equation results.

$$\int_{p_1}^{p_2} V\mathrm{d}p+\frac{\Delta v^2}{2g_c}+\frac{g}{g_c}\Delta Z=-E_\ell. \quad (4)$$

If flow is isothermal and the fluid is incompressible, Eq. 4 may be simplified to

$$\frac{\Delta p}{\rho}+\frac{\Delta(v^2)}{2g_c}+\frac{g}{g_c}\Delta Z=-E_\ell, \quad (5)$$

where ρ=density.

The dimensions of the energy terms in Eq. 5 are energy per unit mass of fluid, such as foot-pounds per pound. Quite often the force term is canceled (incorrectly) with that of the mass term resulting in the dimensions of length as of a column of fluid. For this reason, these terms frequently are referred to as "head," such as feet of the fluid. For most practical cases, the ratio g/g_c is essentially unity. Although the terms in Eq. 5 are sometimes expressed as feet of fluid, no serious error is involved. In fact, one can derive a very similar expression where the terms are expressed in feet of "head."

Eqs. 4 and 5 are the energy relationships that provide the basis for the computational methods of the sections to follow.

Irreversibility Losses. The use of Eqs. 4 and 5 requires a knowledge of E_ℓ, the term that accounts for irreversibilities (such as friction) in the system. The term E_ℓ can be expressed as follows[1]:

$$E_\ell=\frac{fLv^2}{2g_c d}, \quad (6)$$

where f commonly is referred to as a friction factor, L is length, and d is pipe diameter. The friction factor, f, usually is expressed in terms of the physical variables of the system by correlations of experimental data.

For single-phase flow, the dimensionless friction factor, f, has been correlated in terms of the dimensionless Reynolds number $dv\rho/\mu$ with μ being viscosity. A relationship is also suggested by application of dimensional analysis to the variables involved. In either case the result is

$$f=F_1\frac{(dv\rho)}{\mu}, \quad (7)$$

where F_1 is a function of Reynolds number.

Eq. 7 has been the basis for correlation of considerable experimental data for single-phase flow over the past years. Eqs. 5, 6, and 7 have been adapted to multiphase flow. Consideration of the character of pipe surfaces as absolute roughness, ϵ (that is, the distance from peaks to valleys in pipe-wall irregularities), which may be expressed as a dimensionless relative roughness factor, ϵ/d, has led to improvements in correlations of single-phase flow experimental data

$$f=F_2\left[\left(\frac{dv\rho}{\mu}\right)\left(\frac{\epsilon}{d}\right)\right], \quad (8)$$

where F_2 is a function of Reynolds number and relative roughness.

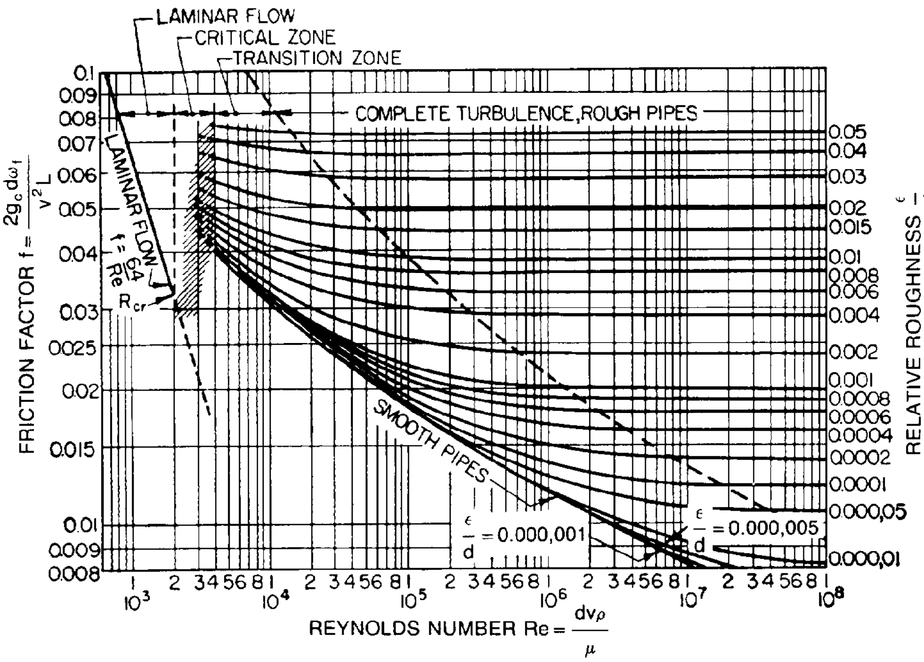

Fig. 34.2—Friction factor as a function of Reynolds number with relative roughness as a parameter.

Fig. 34.2 shows the correlation for single-phase flow according to Eq. 8.[2] Similar plots are found in the literature in which other friction factors are plotted as a function of Reynolds number. Care must be taken to avoid confusion, as the same name and symbol are used for various multiples of f as plotted in Fig. 34.2

The laminar-flow region, which extends up to a Reynolds number of 2,000, is represented by a straight-line relationship $f=64/N_{Re}$ on Fig. 34.2. Between 2,000 and 4,000, flow is unstable. Above 4,000, turbulence prevails and the influence of the physical properties decreases as the Reynolds number increases. In fact, it is shown that at very high Reynolds numbers the friction factor depends solely on the relative roughness factor ϵ/d.

The preceding theoretical discussion concerning irreversibility losses is based on considerations involving single-phase flow. Nevertheless, the material presented will provide a basis for considerations involving both single- and multiphase flow that appear in the following sections.

Static Fluids

Many wellbore problems are associated with static-fluid columns, either oil, water, or gas, or combinations thereof. In the case of static-fluid columns, Eq. 4 is applicable in general and reduces to

$$\int_{p_1}^{p_2} V dp + \frac{g}{g_c}\Delta Z = 0 \quad \text{.......................(9)}$$

or

$$\int_{p_1}^{p_2} \frac{dp}{\rho} + \frac{g}{g_c}\Delta Z = 0, \quad \text{.......................(10)}$$

since $v^2/2g_c$ and E_ℓ are equal to zero. Since g/g_c is assumed to be unity,

$$\int_{p_1}^{p_2} \frac{dp}{\rho} + \Delta Z = 0. \quad \text{.........................(11)}$$

For the case of a static-liquid column, it is usually satisfactory to use an average density for the column of liquid. Eq. 11 then can be expressed in the more convenient and familiar form as

$$\Delta p = \rho \Delta Z. \quad \text{..............................(12)}$$

The preceding equations will provide a basis for the calculation procedures of the following sections for static-fluid columns.

Producing Wells

Gas Wells

Calculation of Static Bottomhole Pressures (BHP's). Static BHP's are used to determine the deliverability of gas wells (backpressure curve) and to develop reservoir information for predicting reservoir performance and deliverability. Several methods for calculating static BHP's have appeared in the literature.[3-6] The methods differ primarily as a result of the assumptions made. All start with Eq. 9 assuming g/g_c is unity for a static column:

$$\int_{p_1}^{p_2} V dp + \Delta Z = 0.$$

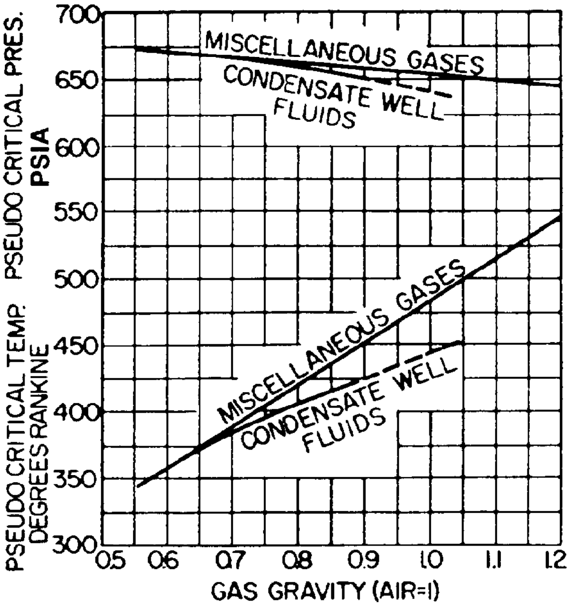

Fig. 34.3—Pseudocritical properties of condensate well fluids and miscellaneous natural gases.

If the column is vertical, $\Delta Z = L$, where L is the length of the pipe string, and Eq. 9 can be put in the form

$$\int_{p_2}^{p_1} V \mathrm{d}p = L. \qquad (13)$$

If the column is not vertical, but inclined with the vertical by an angle θ,

$$\Delta Z = L \cos\theta$$

and again using L, Eq. 9 becomes

$$\int_{p_2}^{p_1} V \mathrm{d}p = L \sin\theta. \qquad (14)$$

Subsequently, only the vertical column will be considered and Eq. 13 will be used. Since

$$V = \frac{zRT}{Mp}, \qquad (15)$$

where

z = compressibility factor,
R = gas constant, and
M = molecular weight,

Eq. 13, upon substitution, becomes

$$\int_{p_2}^{p_1} \frac{zRT}{M} \frac{\mathrm{d}p}{p} = L. \qquad (16)$$

For a particular gas, R/M, which is equal to $53.241/\gamma_g$ where γ_g is the gas gravity (air=1.0), is a constant. Therefore, Eq. 16 can be simplified to

$$\frac{53.241}{\gamma_g} \int_{p_2}^{p_1} zT \frac{\mathrm{d}p}{p} = L. \qquad (17)$$

It is at this point where certain assumptions are made and calculation procedures differ. Assumptions are made in regard to z and T.

For any calculation procedure, four "surface" properties must be known: well-effluent composition, well depth, wellhead pressure, and well temperature. The gas composition is used to calculate the pseudocritical properties p_{pc} and T_{pc} of the gas, from which is estimated the value of the compressibility factor z used in the calculations. Quite often, gas composition is not available and gas gravity must be used to estimate the pseudocritical properties (Fig. 34.3).[4]

A recommended method assumes constant and average temperature $\bar{T}$ and allows z to vary with pressure. With temperature being constant, Eq. 17 becomes

$$\frac{53.241\bar{T}}{\gamma_g} \int_{p_2}^{p_1} \frac{z}{p} \mathrm{d}p = L. \qquad (18)$$

The method using Eq. 18 was suggested by Fowler.[3] Poettmann[4] made the solution of Eq. 18 practical by presenting tables of the function

$$\int_{0.2}^{p_{pr}} \frac{z}{p_{pr}} \mathrm{d}p_{pr}$$

in terms of p_{pr} and T_{pr}. The tables are presented here as Table 34.1.

It can be shown that

$$\int_{p_2}^{p_1} \frac{z}{p} \mathrm{d}p = \int_{(p_{pr})_2}^{(p_{pr})_1} \frac{z}{p_{pr}} \mathrm{d}p_{pr} = \int_{0.2}^{(p_{pr})_1} \frac{z}{p_{pr}} \mathrm{d}p_{pr}$$

$$- \int_{0.2}^{(p_{pr})_2} \frac{z}{p_{pr}} \mathrm{d}p_{pr}. \qquad (19)$$

An advantage of this method is that it is *a direct method of calculating BHP*. No trial and error is involved. In terms of p_{pr} and T_{pr} Eq. 18 becomes

$$L = \frac{53.241\bar{T}}{\gamma_g} \left[\int_{0.2}^{(p_{pr})_1} \frac{z}{p_{pr}} \mathrm{d}p_{pr} - \int_{0.2}^{(p_{pr})_2} \frac{z}{p_{pr}} \mathrm{d}p_{pr} \right]. \qquad (20)$$

By rearranging,

$$\int_{0.2}^{(p_{pr})_1} \frac{z}{p_{pr}} \mathrm{d}p_{pr} = \frac{L\gamma_g}{53.241\bar{T}} + \int_{0.2}^{(p_{pr})_2} \frac{z}{p_{pr}} \mathrm{d}p_{pr}. \qquad (21)$$

Eq. 21 permits a direct solution for the static BHP.

TABLE 34.1—VALUES OF $\int_{0.2}^{p_{pr}} \frac{z}{p_{pr}} dp_{pr}$

Pseudoreduced Pressure p_{pr}	Pseudoreduced Temperature, T_{pr}: 1.05	1.10	1.15	1.20	1.25	1.30	1.35	1.40	1.45	1.50	1.60	1.70	1.80	1.90
0.2	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0.3	0.350	0.350	0.350	0.350	0.350	0.350	0.350	0.350	0.350	0.350	0.350	0.350	0.350	0.350
0.4	0.615	0.619	0.623	0.626	0.628	0.630	0.632	0.633	0.634	0.635	0.636	0.637	0.638	0.639
0.5	0.805	0.816	0.826	0.834	0.839	0.844	0.848	0.851	0.854	0.856	0.860	0.862	0.864	0.866
0.6	0.955	0.971	0.985	0.998	1.011	1.022	1.032	1.040	1.045	1.048	1.049	1.049	1.050	1.050
0.7	1.078	1.100	1.124	1.145	1.162	1.178	1.190	1.199	1.203	1.207	1.210	1.211	1.213	1.214
0.8	1.175	1.207	1.239	1.264	1.285	1.300	1.313	1.322	1.332	1.340	1.347	1.352	1.357	1.359
0.9	1.256	1.300	1.335	1.365	1.386	1.403	1.417	1.429	1.440	1.450	1.462	1.472	1.480	1.485
1.0	1.327	1.375	1.420	1.455	1.479	1.500	1.415	1.530	1.541	1.551	1.568	1.580	1.590	1.598
1.1	1.380	1.438	1.435	1.528	1.552	1.573	1.591	1.606	1.616	1.631	1.653	1.667	1.676	1.684
1.2	1.433	1.500	1.550	1.600	1.625	1.645	1.666	1.682	1.690	1.710	1.737	1.753	1.761	1.770
1.3	1.463	1.545	1.602	1.657	1.684	1.709	1.731	1.746	1.758	1.779	1.810	1.828	1.836	1.845
1.4	1.492	1.590	1.654	1.713	1.742	1.772	1.795	1.810	1.825	1.847	1.882	1.903	1.911	1.920
1.5	1.510	1.620	1.690	1.757	1.791	1.824	1.848	1.867	1.884	1.906	1.938	1.962	1.973	1.984
1.6	1.527	1.649	1.726	1.800	1.839	1.875	1.900	1.923	1.943	1.964	1.993	2.021	2.035	2.047
1.7	1.544	1.670	1.754	1.834	1.876	1.917	1.943	1.969	1.991	2.012	2.043	2.072	2.089	2.102
1.8	1.560	1.690	1.782	1.867	1.913	1.958	1.985	2.014	2.038	2.060	2.093	2.123	2.142	2.157
1.9	1.575	1.708	1.808	1.896	1.944	1.993	2.022	2.054	2.079	2.100	2.136	2.165	2.187	2.204
2.0	1.590	1.725	1.833	1.924	1.975	2.027	2.059	2.093	2.119	2.140	2.178	2.207	2.231	2.250
2.1	1.604	1.743	1.854	1.947	2.003	2.057	2.092	2.126	2.153	2.176	2.215	2.248	2.272	2.292
2.2	1.617	1.761	1.876	1.971	2.031	2.086	2.125	2.160	2.187	2.212	2.252	2.288	2.313	2.334
2.3	1.631	1.779	1.897	1.994	2.059	2.116	2.157	2.193	2.222	2.249	2.288	2.329	2.354	2.375
2.4	1.644	1.797	1.919	2.018	2.087	2.145	2.190	2.227	2.256	2.285	2.325	2.369	2.395	2.417
2.5	1.658	1.815	1.940	2.041	2.115	2.175	2.223	2.260	2.290	2.321	2.362	2.410	2.436	2.459
2.6	1.672	1.830	1.958	2.061	2.137	2.198	2.249	2.288	2.318	2.350	2.392	2.442	2.469	2.492
2.7	1.685	1.845	1.976	2.081	2.159	2.221	2.275	2.316	2.347	2.379	2.423	2.474	2.502	2.525
2.8	1.699	1.860	1.994	2.101	2.180	2.245	2.302	2.344	2.375	2.407	2.453	2.506	2.534	2.557
2.9	1.712	1.875	2.012	2.121	2.202	2.268	2.328	2.372	2.404	2.436	2.484	2.538	2.567	2.590
3.0	1.726	1.890	2.030	2.140	2.224	2.291	2.354	2.400	2.432	2.465	2.514	2.570	2.600	2.623
3.1	1.740	1.904	2.046	2.157	2.243	2.311	2.376	2.423	2.455	2.489	2.540	2.597	2.628	2.652
3.2	1.754	1.918	2.062	2.175	2.261	2.331	2.397	2.446	2.478	2.512	2.565	2.623	2.657	2.681
3.3	1.767	1.932	2.078	2.192	2.280	2.350	2.419	2.469	2.502	2.536	2.591	2.650	2.685	2.709
3.4	1.781	1.946	2.094	2.210	2.298	2.370	2.440	2.492	2.525	2.559	2.616	2.676	2.714	2.738
3.5	1.795	1.960	2.110	2.227	2.317	2.390	2.462	2.515	2.548	2.583	2.642	2.703	2.742	2.767
3.6	1.808	1.974	2.125	2.243	2.333	2.407	2.480	2.535	2.568	2.603	2.664	2.726	2.766	2.792
3.7	1.822	1.988	2.140	2.259	2.349	2.424	2.498	2.556	2.588	2.624	2.686	2.748	2.791	2.817
3.8	1.835	2.002	2.155	2.275	2.365	2.440	2.517	2.576	2.609	2.644	2.708	2.771	2.815	2.843
3.9	1.849	2.016	2.170	2.291	2.381	2.457	2.535	2.597	2.629	2.665	2.730	2.793	2.840	2.868
4.0	1.862	2.030	2.186	2.306	2.397	2.474	2.553	2.617	2.649	2.685	2.752	2.816	2.864	2.893
4.1	1.875	2.044	2.201	2.321	2.413	2.490	2.569	2.634	2.667	2.703	2.771	2.836	2.885	2.915
4.2	1.889	2.058	2.216	2.336	2.429	2.506	2.586	2.651	2.685	2.721	2.789	2.856	2.907	2.937
4.3	1.902	2.073	2.230	2.351	2.444	2.523	2.602	2.669	2.702	2.740	2.808	2.875	2.928	2.958
4.4	1.916	2.087	2.245	2.366	2.460	2.539	2.619	2.686	2.720	2.758	2.826	2.895	2.950	2.980
4.5	1.929	2.101	2.260	2.381	2.476	2.555	2.635	2.703	2.738	2.776	2.845	2.915	2.971	3.002
4.6	1.942	2.115	2.274	2.395	2.491	2.570	2.651	2.719	2.754	2.793	2.863	2.933	2.990	3.022
4.7	1.955	2.128	2.298	2.409	2.507	2.586	2.666	2.735	2.770	2.810	2.881	2.952	3.009	3.041
4.8	1.969	2.142	2.301	2.423	2.522	2.601	2.682	2.752	2.786	2.826	2.899	2.970	3.027	3.061
4.9	1.982	2.155	2.315	2.437	2.538	2.617	2.697	2.768	2.802	2.843	2.917	2.989	3.046	3.080
5.0	1.995	2.169	2.329	2.451	2.553	2.632	2.713	2.784	2.818	2.860	2.935	3.007	3.065	3.100
5.1	2.009	2.183	2.342	2.465	2.567	2.646	2.728	2.799	2.834	2.876	2.952	3.024	3.082	3.118
5.2	2.024	2.197	2.355	2.479	2.581	2.661	2.743	2.814	2.850	2.892	2.968	3.042	3.099	3.136
5.3	2.038	2.210	2.369	2.492	2.595	2.675	2.758	2.830	2.865	2.908	2.985	3.059	3.117	3.153
5.4	2.053	2.224	2.382	2.506	2.609	2.690	2.773	2.845	2.881	2.924	3.001	3.077	3.134	3.171
5.5	2.067	2.238	2.395	2.520	2.623	2.704	2.788	2.860	2.897	2.940	3.018	3.094	3.151	3.189
5.6	2.079	2.251	2.408	2.533	2.636	2.718	2.801	2.874	2.912	2.955	3.037	3.110	3.168	3.206
5.7	2.091	2.264	2.421	2.547	2.650	2.731	2.815	2.888	2.926	2.970	3.049	3.125	3.185	3.224
5.8	2.102	2.277	2.435	2.560	2.663	2.745	2.828	2.902	2.941	2.985	3.065	3.141	3.201	3.241
5.9	2.114	2.290	2.448	2.574	2.677	2.758	2.842	2.916	2.955	3.000	3.080	3.156	3.218	3.259
6.0	2.126	2.303	2.461	2.587	2.690	2.772	2.855	2.930	2.970	3.015	3.096	3.172	3.235	3.276

TABLE 34.1—VALUES OF $\int_{0.2}^{p_{pr}} \frac{z}{p_{pr}} dp_{pr}$ (continued)

Pseudoreduced Pressure p_{pr}	Pseudoreduced Temperature, T_{pr} 1.05	1.10	1.15	1.20	1.25	1.30	1.35	Pseudoreduced Pressure p_{pr}	Pseudoreduced Temperature, T_{pr} 1.40	1.45	1.50	1.60	1.70	1.80	1.90
6.1	2.139	2.316	2.474	2.600	2.703	2.785	2.869	6.1	2.943	2.984	3.029	3.111	3.187	3.250	3.292
6.2	2.152	2.328	2.486	2.612	2.716	2.799	2.882	6.2	2.956	2.997	3.043	3.125	3.202	3.266	3.308
6.3	2.165	2.341	2.499	2.625	2.729	2.812	2.896	6.3	2.970	3.011	3.056	3.140	3.218	3.281	3.323
6.4	2.178	2.353	2.511	2.637	2.742	2.826	2.909	6.4	2.983	3.024	3.070	3.154	3.233	3.297	3.339
6.5	2.191	2.366	2.524	2.650	2.755	2.839	2.923	6.5	2.996	3.038	3.084	3.169	3.248	3.312	3.355
6.6	2.204	2.379	2.536	2.662	2.768	2.852	2.936	6.6	3.009	3.051	3.098	3.183	3.262	3.327	3.370
6.7	2.217	2.391	2.548	2.675	2.781	2.864	2.949	6.7	3.022	3.064	3.112	3.197	3.276	3.341	3.385
6.8	2.229	2.404	2.560	2.687	2.794	2.877	2.963	6.8	3.034	3.077	3.126	3.210	3.291	3.356	3.399
6.9	2.242	2.416	2.572	2.700	2.807	2.889	2.976	6.9	3.047	3.090	3.140	3.224	3.305	3.370	3.414
7.0	2.255	2.429	2.584	2.712	2.870	2.902	2.989	7.0	3.060	3.103	3.154	3.238	3.319	3.385	3.429
7.1	2.268	2.442	2.597	2.724	2.832	2.915	3.002	7.1	3.073	3.116	3.167	3.251	3.332	3.399	3.443
7.2	2.281	2.454	2.609	2.737	2.844	2.928	3.014	7.2	3.085	3.129	3.180	3.264	3.345	3.413	3.457
7.3	2.294	2.467	2.622	2.749	2.856	2.941	3.027	7.3	3.098	3.141	3.194	3.278	3.359	3.427	3.472
7.4	2.307	2.479	2.634	2.762	2.868	2.954	3.039	7.4	3.110	3.154	3.207	3.291	3.372	3.441	3.486
7.5	2.320	2.492	2.647	2.774	2.880	2.967	3.052	7.5	3.123	3.167	3.220	3.304	3.385	3.455	3.500
7.6	2.333	2.505	2.660	2.786	2.892	2.979	3.065	7.6	3.135	3.180	3.233	3.317	3.398	3.468	3.514
7.7	2.346	2.517	2.672	2.799	2.904	2.991	3.077	7.7	3.147	3.192	3.246	3.330	3.411	3.482	3.528
7.8	2.359	2.530	2.685	2.811	2.916	3.003	3.090	7.8	3.160	3.205	3.260	3.344	3.424	3.495	3.541
7.9	2.372	2.542	2.697	2.824	2.928	3.015	3.102	7.9	3.172	3.217	3.274	3.357	3.437	3.509	3.555
8.0	2.385	2.555	2.710	2.836	2.940	3.027	3.115	8.0	3.184	3.230	3.287	3.370	3.450	3.522	3.569
8.1	2.398	2.568	2.723	2.848	2.952	3.039	3.127	8.1	3.197	3.242	3.299	3.382	3.462	3.534	3.581
8.2	2.411	2.580	2.736	2.861	2.964	3.051	3.139	8.2	3.209	3.254	3.311	3.394	3.474	3.546	3.594
8.3	2.424	2.593	2.748	2.873	2.977	3.064	3.151	8.3	3.222	3.266	3.323	3.407	3.486	3.559	3.606
8.4	2.437	2.605	2.761	2.886	2.989	3.076	3.163	8.4	3.234	3.278	3.335	3.419	3.498	3.571	3.619
8.5	2.450	2.618	2.774	2.898	3.001	3.088	3.175	8.5	3.247	3.290	3.347	3.431	3.510	3.583	3.631
8.6	2.462	2.631	2.787	2.910	3.013	3.100	3.187	8.6	3.259	3.302	3.359	3.443	3.523	3.595	3.643
8.7	2.475	2.643	2.799	2.923	3.025	3.112	3.199	8.7	3.270	3.315	3.370	3.456	3.535	3.607	3.655
8.8	2.487	2.656	2.812	2.935	3.038	3.124	3.211	8.8	3.282	3.327	3.382	3.468	3.548	3.619	3.666
8.9	2.500	2.668	2.824	2.948	3.050	3.136	3.223	8.9	3.293	3.340	3.393	3.481	3.560	3.631	3.678
9.0	2.512	2.681	2.837	2.960	3.062	3.148	3.235	9.0	3.305	3.352	3.405	3.493	3.573	3.643	3.690
9.1	2.524	2.693	2.849	2.972	3.074	3.159	3.246	9.1	3.317	3.364	3.417	3.505	3.585	3.655	3.702
9.2	2.536	2.706	2.861	2.985	3.085	3.170	3.257	9.2	3.329	3.376	3.429	3.517	3.597	3.667	3.714
9.3	2.549	2.718	2.872	2.997	3.097	3.182	3.268	9.3	3.340	3.388	3.440	3.530	3.608	3.678	3.725
9.4	2.561	2.731	2.884	3.010	3.108	3.193	3.279	9.4	3.352	3.400	3.452	3.542	3.620	3.690	3.737
9.5	2.573	2.743	2.896	3.022	3.120	3.204	3.290	9.5	3.364	3.412	3.464	3.554	3.632	3.702	3.749
9.6	2.585	2.755	2.908	3.034	3.131	3.216	3.302	9.6	3.376	3.424	3.475	3.565	3.644	3.713	3.760
9.7	2.597	2.767	2.919	3.045	3.142	3.228	3.314	9.7	3.388	3.435	3.487	3.576	3.656	3.724	3.772
9.8	2.610	2.780	2.931	3.057	3.153	3.239	3.326	9.8	3.399	3.447	3.498	3.588	3.667	3.736	3.783
9.9	2.622	2.792	2.942	3.068	3.164	3.251	3.338	9.9	3.411	3.458	3.510	3.599	3.679	3.747	3.795
10.0	2.634	2.804	2.954	3.080	3.175	3.263	3.350	10.0	3.423	3.470	3.521	3.610	3.691	3.758	3.806
10.1	2.646	2.816	2.966	3.092	3.187	3.274	3.361	10.1	3.434	3.482	3.532	3.622	3.702	3.769	3.817
10.2	2.658	2.828	2.978	3.103	3.199	3.286	3.372	10.2	3.446	3.494	3.544	3.633	3.714	3.780	3.828
10.3	2.671	2.840	2.989	3.115	3.211	3.297	3.382	10.3	3.457	3.506	3.555	3.645	3.725	3.790	3.840
10.4	2.683	2.852	3.001	3.126	3.223	3.309	3.393	10.4	3.469	3.518	3.567	3.656	3.737	3.801	3.851
10.5	2.695	2.864	3.013	3.138	3.235	3.320	3.404	10.5	3.480	3.530	3.578	3.668	3.748	3.812	3.862
10.6	2.707	2.876	3.025	3.150	3.246	3.332	3.416	10.6	3.492	3.541	3.588	3.679	3.758	3.823	3.873
10.7	2.719	2.888	3.037	3.161	3.258	3.343	3.428	10.7	3.504	3.552	3.598	3.689	3.769	3.834	3.883
10.8	2.732	2.900	3.048	3.173	3.269	3.355	3.440	10.8	3.515	3.562	3.609	3.700	3.779	3.844	3.894
10.9	2.744	2.912	3.060	3.184	3.281	3.366	3.452	10.9	3.527	3.573	3.619	3.710	3.790	3.855	3.904
11.0	2.756	2.924	3.072	3.196	3.292	3.378	3.464	11.0	3.539	3.584	3.629	3.721	3.800	3.866	3.915
11.1	2.768	2.936	3.084	3.208	3.304	3.389	3.475	11.1	3.551	3.595	3.639	3.732	3.811	3.877	3.926
11.2	2.780	2.948	3.096	3.220	3.315	3.401	3.486	11.2	3.562	3.605	3.650	3.743	3.822	3.888	3.937
11.3	2.793	2.960	3.108	3.231	3.327	3.412	3.497	11.3	3.574	3.616	3.660	3.753	3.832	3.899	3.947
11.4	2.805	2.972	3.120	3.243	3.338	3.424	3.508	11.4	3.585	3.626	3.671	3.764	3.843	3.910	3.958
11.5	2.817	2.984	3.132	3.255	3.350	3.435	3.519	11.5	3.597	3.637	3.681	3.775	3.854	3.921	3.969
11.6	2.829	2.996	3.144	3.267	3.361	3.446	3.529	11.6	3.607	3.648	3.692	3.786	3.865	3.932	3.980
11.7	2.841	3.008	3.156	3.279	3.373	3.456	3.540	11.7	3.617	3.658	3.702	3.797	3.876	3.943	3.991
11.8	2.854	3.020	3.168	3.290	3.384	3.467	3.550	11.8	3.628	3.669	3.713	3.808	3.886	3.955	4.003
11.9	2.866	3.032	3.180	3.302	3.396	3.477	3.561	11.9	3.638	3.679	3.723	3.819	3.897	3.966	4.014
12.0	2.878	3.044	3.192	3.314	3.407	3.488	3.571	12.0	3.648	3.690	3.734	3.830	3.908	3.977	4.025

TABLE 34.1—VALUES OF $\int_{0.2}^{p_{pr}} \frac{z}{p_{pr}} dp_{pr}$ (continued)

Pseudoreduced Pressure p_{pr}	Pseudoreduced Temperature, T_{pr} 2.00	2.20	2.40	2.60	2.80	3.00
0.2	0	0	0	0	0	0
0.3	0.350	0.350	0.350	0.350	0.350	0.350
0.4	0.639	0.640	0.640	0.640	0.640	0.640
0.5	0.867	0.868	0.869	0.869	0.869	0.869
0.6	1.050	1.051	1.051	1.052	1.052	1.052
0.7	1.216	1.218	1.219	1.220	1.220	1.220
0.8	1.360	1.363	1.364	1.364	1.364	1.364
0.9	1.489	1.492	1.494	1.495	1.495	1.495
1.0	1.602	1.607	1.608	1.609	1.610	1.610
1.1	1.691	1.699	1.702	1.706	1.709	1.711
1.2	1.780	1.790	1.795	1.802	1.808	1.812
1.3	1.858	1.868	1.875	1.883	1.890	1.896
1.4	1.935	1.945	1.954	1.964	1.972	1.980
1.5	1.997	2.010	2.019	2.027	2.036	2.045
1.6	2.059	2.074	2.083	2.090	2.100	2.110
1.7	2.116	2.131	2.141	2.148	2.159	2.169
1.8	2.172	2.188	2.198	2.205	2.217	2.227
1.9	2.219	2.237	2.247	2.256	2.267	2.279
2.0	2.265	2.285	2.295	2.307	2.317	2.330
2.1	2.307	2.326	2.337	2.350	2.361	2.375
2.2	2.349	2.366	2.380	2.394	2.404	2.420
2.3	2.391	2.407	2.422	2.437	2.448	2.465
2.4	2.433	2.447	2.465	2.481	2.491	2.510
2.5	2.475	2.488	2.507	2.524	2.535	2.555
2.6	2.508	2.523	2.544	2.562	2.574	2.593
2.7	2.541	2.559	2.581	2.599	2.612	2.630
2.8	2.575	2.594	2.617	2.637	2.651	2.668
2.9	2.608	2.630	2.654	2.674	2.689	2.705
3.0	2.641	2.665	2.691	2.712	2.728	2.743
3.1	2.670	2.694	2.722	2.744	2.759	2.775
3.2	2.700	2.723	2.753	2.775	2.790	2.806
3.3	2.729	2.752	2.783	2.807	2.821	2.838
3.4	2.759	2.781	2.814	2.838	2.852	2.869
3.5	2.788	2.810	2.845	2.870	2.883	2.901
3.6	2.813	2.836	2.872	2.910	2.911	2.929
3.7	2.839	2.862	2.899	2.950	2.938	2.957
3.8	2.864	2.888	2.925	2.990	2.966	2.984
3.9	2.890	2.914	2.952	3.030	2.993	3.012
4.0	2.915	2.940	2.979	3.070	3.021	3.040
4.1	2.938	2.963	3.002	3.081	3.045	3.064
4.2	2.960	2.985	3.025	3.092	3.069	3.088
4.3	2.983	3.008	3.049	3.103	3.094	3.112
4.4	3.005	3.030	3.072	3.114	3.118	3.136
4.5	3.028	3.053	3.095	3.125	3.142	3.160
4.6	3.048	3.074	3.117	3.147	3.164	3.182
4.7	3.068	3.095	3.139	3.168	3.186	3.203
4.8	3.088	3.115	3.161	3.190	3.209	3.225
4.9	3.108	3.136	3.183	3.211	3.231	3.246
5.0	3.128	3.157	3.205	3.233	3.253	3.268
5.1	3.146	3.177	3.225	3.253	3.274	3.288
5.2	3.164	3.196	3.244	3.273	3.295	3.308
5.3	3.182	3.216	3.264	3.294	3.315	3.328
5.4	3.200	3.235	3.283	3.314	3.336	3.348
5.5	3.218	3.255	3.303	3.334	3.357	3.368
5.6	3.235	3.273	3.321	3.352	3.375	3.386
5.7	3.252	3.291	3.339	3.370	3.393	3.405
5.8	3.270	3.309	3.356	3.389	3.412	3.423
5.9	3.287	3.327	3.374	3.407	3.430	3.442
6.0	3.304	3.345	3.392	3.425	3.448	3.460

Pseudoreduced Pressure p_{pr}	Pseudoreduced Temperature, T_{pr} 2.00	2.20	2.40	2.60	2.80	3.00
6.1	3.321	3.362	3.409	3.442	3.466	3.477
6.2	3.337	3.379	3.426	3.459	3.483	3.494
6.3	3.354	3.395	3.443	3.476	3.501	3.511
6.4	3.370	3.412	3.460	3.493	3.518	3.528
6.5	3.387	3.429	3.477	3.510	3.536	3.545
6.6	3.402	3.444	3.493	3.526	3.551	3.561
6.7	3.417	3.459	3.508	3.542	3.567	3.577
6.8	3.432	3.475	3.524	3.557	3.582	3.592
6.9	3.447	3.490	3.539	3.573	3.598	3.608
7.0	3.462	3.505	3.555	3.589	3.613	3.624
7.1	3.477	3.520	3.570	3.604	3.628	3.639
7.2	3.491	3.534	3.584	3.618	3.643	3.654
7.3	3.506	3.549	3.599	3.633	6.659	3.670
7.4	3.520	3.563	3.613	3.647	3.674	3.685
7.5	3.535	3.578	3.628	3.662	3.689	3.700
7.6	3.548	3.591	3.642	3.676	3.703	3.714
7.7	3.562	3.605	3.656	3.690	3.718	3.728
7.8	3.575	3.618	3.670	3.704	3.732	3.742
7.9	3.589	3.632	3.684	3.718	3.747	3.756
8.0	3.602	3.645	3.698	3.732	3.761	3.770
8.1	3.615	3.658	3.711	3.745	3.774	3.783
8.2	3.627	3.671	3.723	3.758	3.788	3.796
8.3	3.640	3.684	3.736	3.771	3.801	3.810
8.4	3.652	3.697	3.748	3.784	3.815	3.823
8.5	3.665	3.710	3.761	3.797	3.828	3.836
8.6	3.677	3.722	3.773	3.810	3.840	3.849
8.7	3.690	3.734	3.786	3.823	3.853	3.862
8.8	3.702	3.746	3.798	3.835	3.865	3.875
8.9	3.715	3.758	3.811	3.848	3.878	3.888
9.0	3.727	3.770	3.823	3.861	3.890	3.901
9.1	3.739	3.782	3.835	3.873	3.902	3.913
9.2	3.750	3.794	3.847	3.885	3.915	3.925
9.3	3.762	3.806	3.859	3.897	3.927	3.938
9.4	3.773	3.818	3.871	3.909	3.940	3.950
9.5	3.785	3.830	3.983	3.921	3.952	3.962
9.6	3.797	3.842	3.895	3.933	3.964	3.974
9.7	3.809	3.854	3.907	3.945	3.976	3.986
9.8	3.820	3.865	3.918	3.957	3.987	3.999
9.9	3.832	3.877	3.930	3.969	3.999	4.011
10.0	3.844	3.889	3.942	3.981	4.011	4.023
10.1	3.855	3.900	3.953	3.992	4.023	4.035
10.2	3.867	3.911	3.965	4.004	4.035	4.046
10.3	3.878	3.923	3.976	4.015	4.046	4.058
10.4	3.890	3.934	3.988	4.027	4.058	4.069
10.5	3.901	3.945	3.999	4.038	4.070	4.081
10.6	3.912	3.956	4.010	4.049	4.081	4.092
10.7	3.923	3.967	4.021	4.060	4.093	4.104
10.8	3.933	3.978	4.031	4.071	4.104	4.115
10.9	3.944	3.989	4.042	4.082	4.116	4.127
11.0	3.955	4.000	4.053	4.093	4.127	4.138
11.1	3.966	4.011	4.064	4.104	4.138	4.149
11.2	3.977	4.022	4.075	4.116	4.150	4.160
11.3	3.988	4.033	4.087	4.127	4.161	4.172
11.4	3.999	4.044	4.098	4.139	4.173	4.183
11.5	4.010	4.055	4.109	4.150	4.184	4.194
11.6	4.022	4.067	4.121	4.161	4.195	4.205
11.7	4.034	4.079	4.132	4.172	4.206	4.216
11.8	4.045	4.090	4.144	4.183	4.217	4.227
11.9	4.057	4.102	4.155	4.194	4.228	4.238
12.0	4.069	4.114	4.167	4.205	4.239	4.249

Example Problem 1.[4] Calculate the static BHP of a gas well having a depth of 5,790 ft; the gas gravity is 0.60, and the pressure at the wellhead is 2,300 psia. The average temperature of the flow string is 117°F.

From Fig. 34.3,

$$T_{pc}=358°\text{R},$$

$$p_{pc}=672 \text{ psia},$$

$$T_{pr}=\frac{T}{T_{pc}}=\frac{117+460}{358}=1.612, \text{ and}$$

$$(p_{pr})_2=\frac{2,300}{672}=3.423.$$

From Table 34.1,

$$\int_{0.2}^{(p_{pr})_2}\frac{z}{p_{pr}}\mathrm{d}p_{pr}=2.629$$

and

$$\frac{L\gamma_g}{53.241\bar{T}}=\frac{(5,790)(0.60)}{(53.241)(577)}=0.113.$$

Therefore, from Eq. 21

$$\int_{0.2}^{(p_{pr})_1}\frac{z}{p_{pr}}\mathrm{d}p_{pr}=2.629+0.113=2.742.$$

From Table 34.1, 2.742 at a T_{pr} of 1.612 corresponds to a p_{pr} of 3.918. Then

$$p=3.918(672)=2,633 \text{ psia}.$$

If temperature is linear with depth,

$$T=aL+b \quad \text{(22)}$$

and

$$\mathrm{d}T=a\ \mathrm{d}L, \quad \text{(23)}$$

where a and b are constants. By substituting Eq. 23 in Eq. 17 and putting in the differential form, the following is obtained:

$$\frac{\mathrm{d}T}{aT}=\frac{53.241z}{\gamma_g}\frac{\mathrm{d}p}{p}. \quad \text{(24)}$$

Integrating,

$$\frac{1}{a}\ln\frac{T_1}{T_2}=\frac{53.241}{\gamma_g}\int_{p_2}^{p_1}z\frac{\mathrm{d}p}{p}. \quad \text{(25)}$$

Since $a=(T_1-T_2)/L$,

$$\frac{L/(T_1-T_2)}{\ln T_1/T_2}=\frac{L}{T_{LM}}=\frac{53.241}{\gamma_g}\int_{p_2}^{p_1}z\frac{\mathrm{d}p}{p}, \quad \text{(26)}$$

then

$$L=\frac{53.241T_{LM}}{\gamma_g}\int_{p_2}^{p_1}\frac{z}{p}\mathrm{d}p, \quad \text{(27)}$$

where

$$T_{LM}=\frac{T_1-T_2}{\ln T_1/T_2};$$

T_1 and T_2 are, respectively, bottomhole and wellhead temperatures. It can be seen that Eq. 27 differs from Eq. 18 only in that here a log mean temperature T_{LM} is used, whereas Eq. 18 uses the arithmetic average temperature, $\bar{T}$.

Referring to the example as an illustration of the calculation procedure using the log-mean-temperature concept, T_{LM} merely is substituted for $\bar{T}$.

Another method of calculating static BHP in gas wells is based on the following equation.

$$p_1=p_2e^{0.01877\gamma_g L/(\bar{T}\bar{z})}. \quad \text{(28)}$$

Eq. 28 can be derived from Eq. 17 if an arithmetic average temperature $\bar{T}$ and an arithmetic average compressibility factor $\bar{z}$ are used. *The method using Eq. 28 is a trial-and-error procedure.* Values of p_1 are assumed to obtain a value of $\bar{z}$. p_1 then is calculated. The procedure is repeated until the values of p_1 are in agreement.

Example Problem 2. (Data used are from Ref. 5.) Given:

Well A

p_2 = 2,600 psia,
γ_g = 0.744,
L = 7,500 ft,
$\bar{T}$ = 152.5||F=612.5||R,
p_{pc} = 663.8 psia (from Fig. 34.3), and
T_{pc} = 385.6°R (from Fig. 34.3).

First Trial. Assume:

p_1 = 3,100 psia,
$\bar{p}$ = 2,850 psia,
p_{pr} = 2,850/663.8=4.30,
T_{pr} = 612.5/385.6=1.59, and
$\bar{z}$ = 0.820.

Therefore,

$$e^{0.01877\gamma_g L/(\bar{T}\bar{z})}=e^{[(0.01877)(0.744)(7,500)]/[(612.5)(0.820)]}=e^{0.2082}=1.2239.$$

$$p_1=(2,600)(1.2239)=3,182 \text{ calculated}.$$

Second Trial. Assume:

$$\begin{aligned} p_1 &= 3{,}182 \text{ psia,} \\ \bar{p} &= 2{,}891 \text{ psia,} \\ p_{pr} &= 2{,}891/663.8=4.36, \\ T_{pr} &= 1.59, \text{ and} \\ \bar{z} &= 0.821. \end{aligned}$$

Therefore,

$$\begin{aligned} p_1 &= (2{,}600)(e^{0.2082}), \\ &= (2{,}600)(1.2239) \\ &= 3{,}182 \text{ psia calculated check.} \end{aligned}$$

Measured pressure at 7,500 ft equals 3,193 psia.

Calculation of Flowing BHP's: Flow in Tubing. Flowing BHP (BHFP) of a gas well when used with the known static formation pressure provides the basis for evaluating the well's deliverability. In wells that produce through tubing and have no packer, the static column of gas in the tubing-casing annulus is exposed to the producing formation. In this case, BHFP, or sandface pressure, can be determined by the relatively simple procedure of calculating the pressure at the bottom of the static column of gas in the annular space. The preceding section describes this calculation procedure. Where a gas well is equipped with a tubing-casing packer, it becomes necessary to use the flowing-gas column in calculating the BHFP.

Use of the flowing-gas column means that energy changes resulting from frictional effects, as well as the energy differences caused by the compressional effects and potential-energy changes, enter into the calculations.

Several methods have been developed for calculating the pressure drop in flowing-gas columns.[4,6,7] Sukkar and Cornell's method[6] is described in detail. Raghaven and Ramey[8] extended Sukkar and Cornell's method to cover reduced temperatures to 3.0 and reduced pressures to 30. In a subsequent section that deals with gas flow in injection wells, Poettmann's[4] method is described. Poettmann's method can be used for upward flow also.

The basic energy equation, Eq. 3, for any flowing fluid in differential form is

$$V\mathrm{d}p+\frac{v\mathrm{d}v}{g_c}+\frac{g}{g_c}\mathrm{d}Z-\mathrm{d}E_\ell-\mathrm{d}W=0. \quad \text{(29)}$$

Assuming that the kinetic-energy term is small and can be taken as zero, and recognizing that dW, work done by or on the fluid, is zero, Eq. 29 reduces to

$$V\mathrm{d}p+\frac{g}{g_c}\mathrm{d}Z+\mathrm{d}E_\ell=0. \quad \text{(30)}$$

For vertical gas flow, dZ=dL. Since

$$V=\frac{zRT}{Mp} \quad \text{(15)}$$

and

$$\frac{g}{g_c}=1.0$$

and

$$\mathrm{d}E_\ell=\frac{fv^2\mathrm{d}L}{2g_c d}, \quad \text{(31)}$$

Eq. 30, upon substitution, becomes

$$\frac{RzT\mathrm{d}p}{Mp}+\left(1+\frac{fv^2}{2g_c d}\right)\mathrm{d}L=0. \quad \text{(32)}$$

Velocity can be expressed in terms of volumetric flow rate and pipe diameter. Pressure can be expressed in terms of reduced pressure. Substituting these terms in Eq. 32, integrating the equation, and converting to common units results in

$$\int_{(p_{pr})_1}^{(p_{pr})_2}\frac{(z/p_{pr})\mathrm{d}p_{pr}}{1+B(z/p_{pr})^2}=-0.01877\gamma_g\int_{L_1}^{L_2}\frac{\mathrm{d}L}{T} \quad \text{(33)}$$

where

$$\begin{aligned} B &= \frac{667fq_g^{\,2}\bar{T}^2}{d_i^{\,5}p_{pc}^{\,2}}, \\ \gamma_g &= \text{gas gravity (air}=1.0), \\ L &= \text{length of flow string, ft,} \\ T &= \text{temperature, °R,} \\ \bar{T} &= \text{average temperature, °R,} \\ f &= \text{friction factor, dimensionless,} \\ q_g &= \text{flow rate, } 10^6 \text{ cu ft/D referred to 14.65 psia and 60°F,} \\ d_i &= \text{inside diameter of pipe, in.,} \\ p_{pc} &= \text{pseudocritical pressure, psia, and} \\ p_{pr} &= \text{pseudoreduced pressure } p/p_{pc}. \end{aligned}$$

At this point, it is further assumed that temperature is constant at some average value. This permits direct integration of the right side of Eq. 33, as

$$\int_{(p_{pr})_2}^{(p_{pr})_1}\frac{(z/p_{pr})\mathrm{d}p_{pr}}{1+B(z/p_{pr})^2}=\frac{0.01877}{\bar{T}}\gamma_g L, \quad \text{(34)}$$

where the limits of the integral are inverted to change the sign. If the temperature is linear with depth, the use of log mean temperature as the average temperature provides a rigorous solution to the right side of Eq. 34. This use of log mean temperature confines the effect of the assumption of constant temperature to the left side of the equation, where, for practical purposes, it is extremely small. Thus, errors introduced by the assumption of constant temperature are negligible.

(continued on Page 34-23)

TABLE 34.2—EXTENDED SUKKAR-CORNELL INTEGRAL FOR BHP CALCULATION

$$\int_{0.2}^{p_{pr}} \frac{(z/p_{pr})dp_{pr}}{1+B(z/p_{pr})^2}$$

Pseudoreduced temperature for $B = 0.0$

p_{pr}	1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.8	1.9	2.0	2.2	2.4	2.6	2.8	3.0
0.20	0.0000	0.0000	0.0000	0.0000	0.0000	0.000	0.000	0.000	0.000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.50	0.8387	0.8582	0.8719	0.8824	0.8897	0.8966	0.9017	0.9079	0.9082	0.9108	0.9147	0.9177	0.9194	0.9206	0.9218
1.00	1.3774	1.4440	1.4836	1.5129	1.5334	1.5514	1.5654	1.5781	1.5823	1.5889	1.5986	1.6059	1.6111	1.6148	1.6184
1.50	1.6048	1.7373	1.8078	1.8565	1.8911	1.9192	1.9422	1.9609	1.9693	1.9798	1.9951	2.0063	2.0151	2.0211	2.0274
2.00	1.7149	1.9116	2.0157	2.0842	2.1331	2.1709	2.2023	2.2273	2.2397	2.2536	2.2744	2.2893	2.3013	2.3100	2.3184
2.50	1.7995	2.0298	2.1631	2.2507	2.3138	2.3607	2.3996	2.4307	2.4469	2.4641	2.4900	2.5081	2.5234	2.5347	2.5452
3.00	1.8750	2.1255	2.2778	2.3813	2.4570	2.5125	2.5583	2.5947	2.6148	2.6354	2.6654	2.6863	2.7050	2.7189	2.7314
3.50	1.9473	2.2101	2.3746	2.4898	2.5762	2.6390	2.6909	2.7325	2.7561	2.7798	2.8138	2.8382	2.8589	2.8752	2.8896
4.00	2.0178	2.2822	2.4603	2.5845	2.6793	2.7480	2.8052	2.8515	2.8784	2.9050	2.9426	2.9699	2.9928	3.0114	3.0274
4.50	2.0889	2.3622	2.5390	2.6698	2.7715	2.8449	2.9065	2.9569	2.9867	3.0158	3.0571	3.0871	3.1119	3.1322	3.1496
5.00	2.1547	2.4330	2.6128	2.7484	2.8558	2.9330	2.9982	3.0523	3.0645	3.1158	3.1605	3.1930	3.2195	3.2413	3.2597
5.50	2.2214	2.5013	2.6833	2.8222	2.9341	3.0146	3.0828	3.1400	3.1742	3.2074	3.2552	3.2899	3.3178	3.3408	3.3600
6.00	2.2872	2.5577	2.7512	2.8926	3.0079	3.0911	3.1616	3.2215	3.2575	3.2924	3.3428	3.3795	3.4085	3.4325	3.4524
6.50	2.3522	2.6329	2.8171	2.9603	3.0781	3.1635	3.2360	3.2980	3.3355	3.3720	3.4245	3.4629	3.4931	3.5176	3.5381
7.00	2.4165	2.6971	2.8814	3.0258	3.1452	3.2324	3.3065	3.3704	3.4092	3.4470	3.5012	3.5411	3.5722	3.5973	3.6181
7.50	2.4802	2.7602	2.9442	3.0893	3.2100	3.2985	3.3740	3.4393	3.4792	3.5180	3.5738	3.6148	3.5467	3.6723	3.6934
8.00	2.5432	2.8223	3.0058	3.1512	3.2727	3.3623	3.4387	3.5052	3.5460	3.5857	3.6486	3.6847	3.7173	3.7432	3.7646
8.50	2.6057	2.8836	3.0664	3.2118	3.3338	3.4239	3.5012	3.5685	3.6101	3.6504	3.7144	3.7512	3.7844	3.8108	3.8323
9.00	2.6676	2.9441	3.1260	3.2713	3.3934	3.4838	3.5617	3.6297	3.6718	3.7126	3.7775	3.8148	3.8484	3.8750	3.8969
9.50	2.7289	3.0039	3.1847	3.3296	3.4516	3.5422	3.6204	3.6889	3.7315	3.7727	3.6382	3.8760	3.9099	3.9357	3.9588
10.00	2.7896	3.0630	3.2427	3.3870	3.5087	3.5993	3.6776	3.7465	3.7894	3.8308	3.8969	3.9350	3.9690	3.9961	4.0182
10.50	2.8499	3.1215	3.2999	3.4436	3.5647	3.6552	3.7336	3.8026	3.8456	3.8872	3.9538	3.9921	4.0262	4.0533	4.0755
11.00	2.9096	3.1794	3.3565	3.4993	3.6198	3.7100	3.7883	3.8573	3.9004	3.9421	4.0090	4.0473	4.0814	4.1086	4.1309
11.50	2.9690	3.2369	3.4126	3.5543	3.6741	3.7640	3.8420	3.9108	3.9540	3.9958	4.0627	4.1010	4.1351	4.1622	4.1845
12.00	3.0280	3.2940	3.4681	3.6086	3.7277	3.8171	3.8948	3.9634	4.0065	4.0432	4.1150	4.1532	4.1872	4.2143	4.2366
12.50	3.0867	3.3506	3.5231	3.6623	3.7806	3.8694	3.9467	4.0150	4.0579	4.0994	4.1660	4.2041	4.2380	4.2650	4.2872
13.00	3.1452	3.4068	3.5777	3.7154	3.8328	3.9211	3.9977	4.0557	4.1084	4.1495	4.2158	4.2537	4.2875	4.3144	4.3365
13.50	3.2033	3.4627	3.6319	3.7680	3.8644	3.9721	4.0480	4.1155	4.1580	4.1989	4.2845	4.3021	4.3357	4.3625	4.3846
14.00	3.2612	3.5183	3.6857	8.8200	3.9354	4.0224	4.0977	4.1547	4.2067	4.2472	4.3122	4.3494	4.3829	4.4095	4.4316
14.50	3.3189	3.5735	3.7391	3.8716	3.9859	4.0722	4.1400	4.2131	4.2546	4.2947	4.3589	4.3957	4.4289	4.4555	4.4775
15.00	3.3763	3.6285	3.7922	3.9228	4.0349	4.1215	4.1950	4.2609	4.3018	4.3414	4.4047	4.4410	4.4741	4.5005	4.5224
15.50	3.4335	3.6832	3.8450	3.9736	4.0855	4.1702	4.2428	4.3080	4.3483	4.3874	4.4497	4.4855	4.5183	4.5446	4.5663
16.00	3.4906	3.7376	3.8974	4.0240	4.1346	4.2185	4.2900	4.3546	4.3942	4.4327	4.4939	4.5291	4.5617	4.5878	4.6094
16.50	3.5474	3.7919	3.9497	4.0740	4.1833	4.2663	4.3388	4.4007	4.4395	4.4773	4.5374	4.5720	4.6042	4.6302	4.6518
17.00	3.6041	3.8459	4.0016	4.1237	4.2316	4.3138	4.3830	4.4462	4.4843	4.5213	4.5802	4.6141	4.6461	4.6719	4.6933
17.50	3.6606	3.8996	4.0533	4.1731	4.2795	4.3608	4.4289	4.4913	4.5285	4.5648	4.6223	4.6555	4.5872	4.7129	4.7341
18.00	3.7170	3.9532	4.1048	4.2221	4.3271	4.4075	4.4743	4.5359	4.5722	4.6077	4.6638	4.6963	4.7276	4.7532	4.7743
18.50	3.7732	4.0066	4.1560	4.2709	4.3744	4.4538	4.5193	4.5801	4.6154	4.6501	4.7048	4.7365	4.7675	4.7928	4.8138
19.00	3.8293	4.0599	4.2071	4.3195	4.4214	4.4998	4.5640	4.6239	4.6582	4.6921	4.7451	4.7761	4.8067	4.8319	4.8527
19.50	3.8853	4.1129	4.2579	4.3678	4.4681	4.5455	4.6053	4.6574	4.7006	4.7335	4.7850	4.8151	4.8454	4.8704	4.8911
20.00	3.9411	4.1658	4.3086	4.4158	4.5145	4.5909	4.6522	4.7104	4.7425	4.7746	4.8244	4.8536	4.8835	4.9083	4.9288
20.50	3.9969	4.2186	4.3590	4.4636	4.5606	4.6360	4.6959	4.7531	4.7841	4.8152	4.8633	4.8916	4.9211	4.9457	4.9661
21.00	4.0525	4.2712	4.4094	4.5112	4.6065	4.6808	4.7392	4.7955	4.8253	4.8554	4.9017	4.9291	4.9582	4.9827	5.0029
21.50	4.1080	4.3237	4.4595	4.5586	4.6522	4.7254	4.7822	4.8376	4.8662	4.8953	4.9397	4.9662	4.9949	5.0192	5.0392
22.00	4.1634	4.3760	4.5095	4.6058	4.6976	4.7697	4.8250	4.8794	4.9068	4.9348	4.9774	5.0029	5.0311	5.0552	5.0751
22.50	4.2187	4.4282	4.5594	4.6528	4.7428	4.8138	4.8675	4.9209	4.9470	4.9739	5.0146	5.0391	5.0670	5.0908	5.1105
23.00	4.2739	4.4803	4.6091	4.6996	4.7879	4.8577	4.9098	4.9621	4.9869	5.0128	5.0514	5.0750	5.1024	5.1260	5.1455
23.50	4.3291	4.5323	4.6587	4.7463	4.8327	4.9014	4.9518	5.0031	5.0265	5.0513	5.0879	5.1104	5.1374	5.1608	5.1802
24.00	4.3841	4.5842	4.7081	4.7928	4.8773	4.9449	4.9935	5.0438	5.0659	5.0895	5.1241	5.1455	5.1720	5.1953	5.2144
24.50	4.4391	4.6360	4.7575	4.8391	4.9217	4.9882	5.0351	5.0843	5.1050	5.1275	5.1599	5.1803	5.2063	5.2294	5.2483
25.00	4.4940	4.6877	4.8067	4.8853	4.9660	5.0312	5.0764	5.1245	5.1438	5.1651	5.1955	5.2147	5.2403	5.2631	5.2819
25.50	4.5488	4.7392	4.8558	4.9314	5.0101	5.0741	5.1176	5.1646	5.1824	5.2025	5.2307	5.2488	5.2739	5.2965	5.3151
26.00	4.6036	4.7907	4.9048	4.9772	5.0541	5.1169	5.1585	5.2044	5.2208	2.2397	5.2656	5.2826	5.3073	5.3296	5.3480
26.50	4.6583	4.8421	4.9536	5.0230	5.0979	5.1594	5.1993	5.2440	5.2589	5.2766	5.3003	5.3162	5.3403	5.3624	5.3806
27.00	4.7129	4.8934	5.0024	5.0686	5.1415	5.2019	5.2398	5.2834	5.2968	5.3132	5.3347	5.3494	5.3730	5.3950	5.4129
27.50	4.7675	4.9447	5.0511	5.1142	5.1850	5.2441	5.2802	5.3227	5.3345	5.3497	5.3588	5.3823	5.4054	5.4272	5.4450
28.00	4.8220	4.9958	5.0997	5.1595	5.2284	5.2862	5.3204	5.3817	5.3720	5.3859	5.4027	5.4150	5.4376	5.4591	5.4767
28.50	4.8764	5.0469	5.1482	5.2048	5.2716	5.3282	5.3605	5.4006	5.4094	5.4219	5.4363	5.4475	5.4695	5.4908	5.5082
29.00	4.9308	5.0979	5.1966	5.2500	5.3147	5.3700	5.4004	5.4393	5.4465	5.4577	5.4697	5.4796	5.5012	5.5223	5.5394
29.50	4.9851	5.1488	5.2450	5.2950	5.3577	5.4117	5.4401	5.4779	5.4834	5.4933	5.5029	5.5116	5.5326	5.5535	5.5704
30.00	5.0394	5.1997	5.2932	5.3400	5.4005	5.4532	5.4797	5.5163	5.5202	5.5287	5.5359	5.5433	5.5638	5.5844	5.6011

TABLE 34.2—EXTENDED SUKKAR-CORNELL INTEGRAL FOR BHP CALCULATION (continued)

$$\int_{0.2}^{p_{pr}} \frac{(z/p_{pr})dp_{pr}}{1+B(z/p_{pr})^2}$$

Pseudoreduced temperature for $B = 5.0$

p_{pr}	1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.8	1.9	2.0	2.2	2.4	2.6	2.8	3.0
0.20	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.50	0.0226	0.0220	0.0216	0.0214	0.0212	0.0210	0.0209	0.0207	0.0207	0.0206	0.0205	0.0205	0.0204	0.0204	0.0204
1.00	0.1036	0.0983	0.0954	0.0934	0.0921	0.0909	0.0901	0.0894	0.0890	0.0886	0.0881	0.0877	0.0874	0.0871	0.0869
1.50	0.2121	0.2052	0.1995	0.1954	0.1924	0.1901	0.1882	0.1668	0.1859	0.1850	0.1838	0.1829	0.1822	0.1816	0.1811
2.00	0.3002	0.3125	0.3102	0.3066	0.3034	0.3007	0.2983	0.2965	0.2954	0.2943	0.2926	0.2914	0.2904	0.2896	0.2889
2.50	0.3741	0.4046	0.4126	0.4133	0.4124	0.4107	0.4090	0.4076	0.4066	0.4056	0.4041	0.4030	0.4020	0.4012	0.4005
3.00	0.4419	0.4854	0.5032	0.5105	0.5137	0.5144	0.5143	0.5140	0.5138	0.5134	0.5125	0.5118	0.5112	0.5108	0.5103
3.50	0.5074	0.5594	0.5847	0.5983	0.6065	0.6101	0.6123	0.6138	0.6147	0.6152	0.6154	0.6155	0.6155	0.6157	0.6156
4.00	0.5715	0.6291	0.6594	0.6785	0.6915	0.6982	0.7029	0.7064	0.7087	0.7104	0.7121	0.7133	0.7140	0.7149	0.7154
4.50	0.6346	0.6957	0.7294	0.7530	0.7702	0.7797	0.7868	0.7927	0.7964	0.7994	0.8027	0.8051	0.8068	0.8084	0.8094
5.00	0.6966	0.7601	0.7960	0.8229	0.8440	0.8560	0.8653	0.8734	0.8785	0.8827	0.8879	0.8916	0.8941	0.8965	0.8980
5.50	0.7579	0.8225	0.8601	0.8895	0.9138	0.9280	0.9393	0.9493	0.9558	0.9611	0.9682	0.9732	0.9765	0.9795	0.9815
6.00	0.8185	0.8836	0.9222	0.9536	0.9803	0.9965	1.0095	1.0213	1.0289	1.0354	1.0441	1.0504	1.0544	1.0580	1.0604
6.50	0.8784	0.9437	0.9829	1.0156	1.0442	1.0620	1.0764	1.0896	1.0984	1.1060	1.1162	1.1236	1.1284	1.1324	1.1351
7.00	0.9378	1.0030	1.0423	1.0758	1.1058	1.1249	1.1406	1.1552	1.1649	1.1734	1.1848	1.1932	1.1987	1.2031	1.2060
7.50	0.9967	1.0614	1.1005	1.1346	1.1656	1.1857	1.2024	1.2182	1.2286	1.2379	1.2504	1.2597	1.2657	1.2704	1.2737
8.00	1.0551	1.1191	1.1578	1.1921	1.2237	1.2447	1.2621	1.2788	1.2900	1.2999	1.3167	1.3234	1.3299	1.3349	1.3383
8.50	1.1131	1.1761	1.2142	1.2486	1.2805	1.3020	1.3201	1.3374	1.3492	1.3596	1.3773	1.3845	1.3914	1.3967	1.4003
9.00	1.1706	1.2325	1.2698	1.3041	1.3361	1.3579	1.3764	1.3943	1.4066	1.4173	1.4357	1.4434	1.4506	1.4561	1.4599
9.50	1.2275	1.2883	1.3248	1.3587	1.3907	1.4125	1.4313	1.4497	1.4623	1.4733	1.4922	1.5008	1.5077	1.5135	1.5174
10.00	1.2841	1.3435	1.3791	1.4126	1.4443	1.4661	1.4851	1.5037	1.5165	1.5278	1.5472	1.5555	1.5630	1.5689	1.5729
10.50	1.3403	1.3983	1.4328	1.4658	1.4970	1.5187	1.5377	1.5564	1.5694	1.5808	1.6006	1.6090	1.6167	1.6226	1.6267
11.00	1.3961	1.4526	1.4860	1.5182	1.5490	1.5705	1.5894	1.6081	1.6211	1.6326	1.6526	1.6611	1.6687	1.6747	1.6789
11.50	1.4515	1.5065	1.5387	1.5701	1.6002	1.6214	1.6401	1.6587	1.6718	1.6833	1.7034	1.7118	1.7195	1.7254	1.7296
12.00	1.5067	1.5601	1.5910	1.6214	1.6509	1.6717	1.6901	1.7085	1.7215	1.7330	1.7530	1.7613	1.7689	1.7749	1.7790
12.50	1.5616	1.6133	1.6429	1.6721	1.7010	1.7213	1.7393	1.7575	1.7704	1.7817	1.8015	1.8097	1.8172	1.8231	1.8271
13.00	1.6163	1.6662	1.6944	1.7224	1.7505	1.7704	1.7879	1.8057	1.8184	1.8295	1.8489	1.8569	1.8644	1.8701	1.8742
13.50	1.6708	1.7188	1.7456	1.7722	1.7995	1.8188	1.8358	1.8532	1.8656	1.8765	1.8954	1.9032	1.9105	1.9161	1.9201
14.00	1.7250	1.7711	1.7965	1.8216	1.8480	1.8667	1.8830	1.9001	1.9121	1.9227	1.9410	1.9485	1.9556	1.9612	1.9651
14.50	1.7791	1.8232	1.8470	1.8706	1.8960	1.9142	1.9298	1.9463	1.9580	1.9681	1.9858	1.9920	1.9998	2.0053	2.0091
15.00	1.8330	1.8750	1.8973	1.9192	1.9436	1.9612	1.9760	1.9920	2.0032	2.0128	2.0298	2.0364	2.0432	2.0485	2.0523
15.50	1.8867	1.9266	1.9472	1.9675	1.9909	2.0077	2.0217	2.0372	2.0478	2.0570	2.0730	2.0792	2.0857	2.0910	2.0946
16.00	1.9402	1.9780	1.9970	2.0154	2.0377	2.0538	2.0669	2.0818	2.0918	2.1005	2.1155	2.1212	2.1275	2.1326	2.1362
16.50	1.9936	2.0292	2.0465	2.0631	2.0842	2.0996	2.1117	2.1260	2.1353	2.1434	2.1574	2.1626	2.1686	2.1736	2.1770
17.00	2.0469	2.0802	2.0958	2.1104	2.1303	2.1450	2.1561	2.1697	2.1783	2.1858	2.1987	2.2032	2.2090	2.2138	2.2172
17.50	2.1000	2.1311	2.1449	2.1575	2.1762	2.1900	2.2000	2.2131	2.2209	2.2276	2.2394	2.2433	2.2488	2.2535	2.2567
18.00	2.1530	2.1817	2.1937	2.2043	2.2217	2.2347	2.2437	2.2560	2.2630	2.2690	2.2795	2.2828	2.2880	2.2925	2.2956
18.50	2.2059	2.2323	2.2424	2.2509	2.2670	2.2791	2.2869	2.2985	2.3046	2.3100	2.3191	2.3217	2.3266	2.3309	2.3339
19.00	2.2587	2.2826	2.2909	2.2973	2.3120	2.3233	2.3299	2.3407	2.3459	2.3505	2.3582	2.3600	2.3646	2.3688	2.3717
19.50	2.3113	2.3329	2.3393	2.3434	2.3567	2.3671	2.3725	2.3825	2.3868	2.3906	2.3969	2.3979	2.4022	2.4062	2.4089
20.00	2.3639	2.3830	2.3875	2.3893	2.4012	2.4107	2.4148	2.4241	2.4273	2.4303	2.4350	2.4353	2.4392	2.4431	2.4456
20.50	2.4164	2.4329	2.4355	2.4350	2.4455	2.4541	2.4568	2.4653	2.4675	2.4696	2.4728	2.4723	2.4758	2.4795	2.4819
21.00	2.4688	2.4828	2.4834	2.4306	2.4895	2.4972	2.4986	2.5062	2.5074	2.5086	2.5101	2.5088	2.5119	2.5155	2.5177
21.50	2.5210	2.5325	2.5311	2.5259	2.5333	2.5400	2.5401	2.5468	2.5470	2.5472	2.5471	2.5449	2.5477	2.5510	2.5531
22.00	2.5733	2.5822	2.5788	2.5711	2.5770	2.5827	2.5814	2.5872	2.5862	2.5855	2.5837	2.5806	2.5830	2.5861	2.5881
22.50	2.6254	2.6317	2.6263	2.6161	2.6204	2.6252	2.6224	2.6273	2.6252	2.6235	2.6199	2.6159	2.6179	2.6209	2.6226
23.00	2.6774	2.6811	2.6736	2.6610	2.6637	2.6674	2.6632	2.6672	2.6639	2.6612	2.6558	2.6508	2.6524	2.6552	2.6568
23.50	2.7294	2.7304	2.7209	2.7057	2.7068	2.7095	2.7038	2.7068	2.7023	2.6986	2.6913	2.6854	2.5866	2.6892	2.6906
24.00	2.7813	2.7796	2.7680	2.7503	2.7497	2.7514	2.7441	2.7462	2.7405	2.7357	2.7266	2.7197	2.7204	2.7229	2.7241
24.50	2.8332	2.8288	2.8151	2.7947	2.7924	2.7981	2.7843	2.7854	2.7784	2.7726	2.7615	2.7536	2.7540	2.7562	2.7573
25.00	2.8849	2.8778	2.8620	2.8390	2.8351	2.8346	2.8243	2.8244	2.8161	2.8092	2.7961	2.7872	2.7872	2.7892	2.7901
25.50	2.9367	2.9268	2.9088	2.8832	2.8775	2.8760	2.8640	2.8532	2.8536	2.8456	2.8305	2.8205	2.8200	2.8219	2.8226
26.00	2.9883	2.9757	2.9556	2.9272	2.9198	2.9172	2.9037	2.9018	2.8908	2.8818	2.8646	2.8536	2.8526	2.8543	2.8548
26.50	3.0399	3.0245	3.0022	2.9711	2.9620	2.9583	2.9431	2.9402	2.9279	2.9177	2.8985	2.8864	2.8850	2.8864	2.8867
27.00	3.0915	3.0733	3.0488	3.0149	3.0040	2.9993	2.9824	2.9785	2.9648	2.9534	2.9320	2.9189	2.9170	2.9182	2.9184
27.50	3.1429	3.1220	3.0953	3.0586	3.0459	3.0400	3.0215	3.0165	3.0014	2.9889	2.9654	2.9512	2.9488	2.9498	2.9497
28.00	3.1944	3.1706	3.1417	3.1022	3.0877	3.0807	2.0604	3.0544	3.0379	3.0242	2.9985	2.9832	2.9803	2.9811	2.9809
28.50	3.2458	3.2191	3.1880	3.1457	3.1294	3.1212	3.0992	3.0922	3.0742	3.0593	3.0314	3.0149	3.0116	3.0122	3.0117
29.00	3.2971	3.2676	3.2343	3.1891	3.1710	3.1616	3.1379	3.1297	3.1103	3.0942	3.0641	3.0465	3.0426	3.0430	3.0424
29.50	3.3484	3.3160	3.2804	3.2324	3.2124	3.2019	3.1764	3.1672	3.1463	3.1289	3.0966	3.0778	3.0735	3.0736	3.0728
30.00	3.3997	3.3644	3.3265	3.2756	3.2537	3.2421	3.2148	3.2045	3.1821	3.1635	3.1288	3.1089	3.1040	3.1040	3.1029

TABLE 34.2—EXTENDED SUKKAR-CORNELL INTEGRAL FOR BHP CALCULATION (continued)

$$\int_{0.2}^{p_{pr}} \frac{(z/p_{pr})dp_{pr}}{1+B(z/p_{pr})^2}$$

Pseudoreduced temperature for $B = 10.0$

p_{pr}	1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.8	1.9	2.0	2.2	2.4	2.6	2.8	3.0
0.20	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.50	0.0115	0.0112	0.0110	0.0108	0.0107	0.0107	0.0106	0.0105	0.0105	0.0105	0.0104	0.0104	0.0104	0.0103	0.0103
1.00	0.0561	0.0525	0.0507	0.0494	0.0486	0.0479	0.0474	0.0470	0.0468	0.0465	0.0462	0.0460	0.0458	0.0456	0.0455
1.50	0.1292	0.1187	0.1132	0.1098	0.1074	0.1056	0.1041	0.1031	0.1024	0.1018	0.1009	0.1003	0.0997	0.0994	0.0990
2.00	0.2028	0.1968	0.1891	0.1837	0.1797	0.1767	0.1743	0.1725	0.1713	0.1703	0.1687	0.1676	0.1667	0.1660	0.1653
2.50	0.2684	0.2723	0.2677	0.2624	0.2578	0.2543	0.2513	0.2490	0.2475	0.2461	0.2440	0.2426	0.2413	0.2403	0.2394
3.00	0.3300	0.3422	0.3427	0.3399	0.3364	0.3332	0.3302	0.3278	0.3263	0.3248	0.3225	0.3210	0.3195	0.3184	0.3174
3.50	0.3897	0.4080	0.4130	0.4135	0.4123	0.4102	0.4080	0.4061	0.4047	0.4035	0.4014	0.3999	0.3985	0.3974	0.3964
4.00	0.4485	0.4708	0.4793	0.4832	0.4846	0.4841	0.4830	0.4820	0.4812	0.4803	0.4787	0.4776	0.4764	0.4755	0.4746
4.50	0.5065	0.5315	0.5423	0.5492	0.5533	0.5545	0.5547	0.5549	0.5549	0.5546	0.5538	0.5532	0.5523	0.5517	0.5511
5.00	0.5638	0.5904	0.6029	0.6122	0.6189	0.6217	0.6233	0.6248	0.6256	0.6260	0.6262	0.6263	0.6258	0.6256	0.6252
5.50	0.6204	0.6480	0.6617	0.6729	0.6818	0.6861	0.6891	0.6919	0.6934	0.6946	0.6959	0.6967	0.6967	0.6968	0.6967
6.00	0.6765	0.7045	0.7190	0.7316	0.7424	0.7481	0.7522	0.7563	0.7586	0.7605	0.7629	0.7645	0.7650	0.7654	0.7655
6.50	0.7321	0.7602	0.7752	0.7888	0.8010	0.8079	0.8131	0.8182	0.8214	0.8240	0.8273	0.8297	0.8307	0.8314	0.8317
7.00	0.7873	0.8153	0.8304	0.8447	0.8580	0.8659	0.8720	0.8781	0.8819	0.8852	0.8895	0.8925	0.8940	0.8950	0.8955
7.50	0.8421	0.8697	0.8846	0.8994	0.9134	0.9221	0.9290	0.9360	0.9404	0.9443	0.9494	0.9531	0.9550	0.9562	0.9568
8.00	0.8965	0.9236	0.9381	0.9531	0.9676	0.9770	0.9845	0.9921	0.9971	1.0015	1.0092	1.0115	1.0138	1.0152	1.0160
8.50	0.9506	0.9769	0.9909	1.0059	1.0207	1.0305	1.0385	1.0467	1.0522	1.0569	1.0653	1.0681	1.0706	1.0723	1.0732
9.00	1.0043	1.0296	1.0431	1.0580	1.0729	1.0829	1.0912	1.0999	1.1057	1.1108	1.1197	1.1228	1.1256	1.1275	1.1286
9.50	1.0575	1.0819	1.0947	1.1094	1.1242	1.1342	1.1428	1.1518	1.1579	1.1633	1.1726	1.1760	1.1790	1.1810	1.1822
10.00	1.1104	1.1338	1.1458	1.1601	1.1747	1.1847	1.1935	1.2027	1.2090	1.2145	1.2242	1.2278	1.2309	1.2331	1.2343
10.50	1.1630	1.1852	1.1964	1.2102	1.2245	1.2344	1.2432	1.2525	1.2589	1.2645	1.2746	1.2783	1.2814	1.2836	1.2850
11.00	1.2153	1.2363	1.2466	1.2598	1.2736	1.2834	1.2920	1.3013	1.3078	1.3135	1.3238	1.3275	1.3307	1.3329	1.3343
11.50	1.2674	1.2871	1.2964	1.3089	1.3222	1.3317	1.3402	1.3494	1.3559	1.3616	1.3719	1.3756	1.3788	1.3810	1.3824
12.00	1.3192	1.3376	1.3458	1.3574	1.3702	1.3794	1.3876	1.3967	1.4032	1.4088	1.4190	1.4227	1.4258	1.4280	1.4294
12.50	1.3708	1.3877	1.3949	1.4056	1.4178	1.4266	1.4345	1.4433	1.4497	1.4552	1.4653	1.4688	1.4719	1.4740	1.4753
13.00	1.4222	1.4377	1.4437	1.4533	1.4649	1.4733	1.4807	1.4893	1.4955	1.5008	1.5106	1.5140	1.5169	1.5139	1.5202
13.50	1.4734	1.4873	1.4921	1.5006	1.5115	1.5194	1.5264	1.5346	1.5406	1.5457	1.5551	1.5582	1.5611	1.5630	1.5642
14.00	1.5244	1.5368	1.5403	1.5476	1.5577	1.5652	1.5716	1.5794	1.5851	1.5899	1.5988	1.6016	1.6043	1.6062	1.6074
14.50	1.5753	1.5860	1.5883	1.5942	1.6035	1.6104	1.6163	1.6237	1.6290	1.6335	1.6417	1.6443	1.6468	1.6486	1.6497
15.00	1.6261	1.6351	1.6360	1.6405	1.6490	1.6553	1.6605	1.6575	1.6723	1.6764	1.6840	1.6862	1.6885	1.6902	1.6912
15.50	1.6767	1.6839	1.6835	1.6865	1.6941	1.6999	1.7043	1.7108	1.7151	1.7811	1.7256	1.7274	1.7296	1.7311	1.7320
16.00	1.7271	1.7326	1.7308	1.7323	1.7389	1.7440	1.7477	1.7537	1.7575	1.7607	1.7666	1.7679	1.7699	1.7713	1.7722
16.50	1.7775	1.7811	1.7778	1.7778	1.7834	1.7878	1.7906	1.7961	1.7993	1.8020	1.8070	1.8078	1.8096	1.8109	1.8116
17.00	1.8277	1.8294	1.8247	1.8230	1.8275	1.8314	1.8333	1.8382	1.8407	1.8429	1.8469	1.8472	1.8487	1.8499	1.8505
17.50	1.8778	1.8777	1.8714	1.8680	1.8714	1.8746	1.8756	1.8799	1.8818	1.8833	1.8862	1.8859	1.8872	1.8883	1.8888
18.00	1.9278	1.9257	1.9179	1.9127	1.9151	1.9175	1.9175	1.9212	1.9224	1.9232	1.9251	1.9242	1.9252	1.9261	1.9265
18.50	1.9777	1.9737	1.9643	1.9573	1.9585	1.9602	1.9592	1.9622	1.9626	1.9628	1.9634	1.9619	1.9626	1.9634	1.9637
19.00	2.0276	2.0215	2.0105	2.0017	2.0016	2.0026	2.0005	2.0029	2.0025	2.0020	2.0013	1.9992	1.9996	2.0002	2.0004
19.50	2.0773	2.0692	2.0566	2.0458	2.0446	2.0447	2.0416	2.0433	2.0420	2.0408	2.0388	2.0359	2.0360	2.0365	2.0366
20.00	2.1269	2.1167	2.1026	2.0898	2.0873	2.0867	2.0824	2.0833	2.0812	2.0792	2.0759	2.0723	2.0721	2.0724	2.0723
20.50	2.1765	2.1642	2.1484	2.1336	2.1298	2.1284	2.1229	2.1232	2.1201	2.1173	2.1126	2.1082	2.1077	2.1079	2.1077
21.00	2.2260	2.2116	2.1941	2.1773	2.1722	2.1699	2.1632	2.1627	2.1587	2.1551	2.1489	2.1438	2.1429	2.1429	2.1425
21.50	2.2754	2.2588	2.2396	2.2207	2.2143	2.2112	2.2033	2.2020	2.1970	2.1926	2.1848	2.1789	2.1777	2.1775	2.1770
22.00	2.3248	2.3060	2.2851	2.2641	2.2563	2.2523	2.2432	2.2411	2.2350	2.2298	2.2204	2.2137	2.2121	2.2118	2.2111
22.50	2.3741	2.3531	2.3304	2.3073	2.2981	2.2932	2.2828	2.2799	2.2728	2.2667	2.2557	2.2481	2.2462	2.2457	2.2449
23.00	2.4233	2.4001	2.3757	2.3503	2.3397	2.3340	2.3222	2.3185	2.3103	2.3033	2.2906	2.2822	2.2799	2.2792	2.2783
23.50	2.4725	2.4470	2.4208	2.3932	2.3812	2.3745	2.3615	2.3569	2.3476	2.3397	2.3253	2.3160	2.3133	2.3124	2.3113
24.00	2.5216	2.4938	2.4659	2.4360	2.4226	2.4149	2.4005	2.3951	2.3847	2.3758	2.3597	2.3494	2.3463	2.3453	2.3440
24.50	2.5706	2.5406	2.5108	2.4787	2.4637	2.4552	2.4394	2.4331	2.4215	2.4117	2.3937	2.3826	2.3791	2.3779	2.3765
25.00	2.6196	2.5873	2.5557	2.5212	2.5048	2.4953	2.4761	2.4709	2.4581	2.4473	2.4275	2.4155	2.4115	2.4102	2.4086
25.50	2.6685	2.6339	2.6005	2.5637	2.5457	2.5353	2.5166	2.5085	2.4946	2.4827	2.4611	2.4481	2.4437	2.4422	2.4404
26.00	2.7174	2.6805	2.6452	2.6060	2.5865	2.5751	2.5550	2.5459	2.5308	2.5179	2.4944	2.4804	2.4756	2.4739	2.4719
26.50	2.7663	2.7269	2.6898	2.6482	2.6272	2.6148	2.5932	2.5832	2.5668	2.5529	2.5275	2.5124	2.5073	2.5053	2.5032
27.00	2.8151	2.7734	2.7343	2.6904	2.6677	2.6543	2.6312	2.6203	2.6027	2.5877	2.5603	2.5443	2.5386	2.5365	2.5342
27.50	2.8638	2.8197	2.7788	2.7324	2.7082	2.6938	2.6691	2.6573	2.6384	2.6223	2.5929	2.5758	2.5698	2.5675	2.5650
28.00	2.9125	2.8660	2.8232	2.7743	2.7485	2.7331	2.7069	2.6941	2.6739	2.6567	2.6253	2.6072	2.6007	2.5982	2.5955
28.50	2.9612	2.9123	2.8675	2.8162	2.7887	2.7723	2.7446	2.7307	2.7092	2.6909	2.6575	2.6383	2.6314	2.6286	2.6258
29.00	3.0098	2.9585	2.9118	2.8579	2.8288	2.8114	2.7821	2.7673	2.7444	2.7250	2.6895	2.6692	2.6618	2.6589	2.6558
29.50	3.0584	3.0046	2.9560	2.8996	2.8689	2.8504	2.8194	2.8036	2.7794	2.7589	2.7212	2.6999	2.6920	2.6889	2.6857
30.00	3.1069	3.0507	3.0001	2.9412	2.9088	2.8892	2.8567	2.8399	2.8143	2.7926	2.7528	2.7304	2.7221	2.7187	2.7153

TABLE 34.2—EXTENDED SUKKAR-CORNELL INTEGRAL FOR BHP CALCULATION (continued)

$$\int_{0.2}^{p_{pr}} \frac{(z/p_{pr})dp_{pr}}{1+B(z/p_{pr})^2}$$

Pseudoreduced temperature for $B = 15.0$

p_{pr}	1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.8	1.9	2.0	2.2	2.4	2.6	2.8	3.0
0.20	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.50	0.0077	0.0075	0.0074	0.0073	0.0072	0.0071	0.0071	0.0071	0.0070	0.0070	0.0070	0.0070	0.0069	0.0069	0.0069
1.00	0.0385	0.0359	0.0345	0.0336	0.0330	0.0325	0.0322	0.0319	0.0317	0.0316	0.0313	0.0311	0.0310	0.0309	0.0308
1.50	0.0939	0.0838	0.0793	0.0765	0.0746	0.0732	0.0721	0.0713	0.0708	0.0703	0.0696	0.0692	0.0687	0.0685	0.0682
2.00	0.1571	0.1453	0.1371	0.1319	0.1282	0.1257	0.1236	0.1220	0.1211	0.1202	0.1189	0.1180	0.1172	0.1167	0.1161
2.50	0.2162	0.2093	0.2008	0.1943	0.1892	0.1857	0.1827	0.1804	0.1790	0.1777	0.1758	0.1745	0.1733	0.1724	0.1716
3.00	0.2725	0.2710	0.2648	0.2587	0.2533	0.2493	0.2458	0.2431	0.2413	0.2397	0.2374	0.2357	0.2342	0.2331	0.2320
3.50	0.3275	0.3302	0.3267	0.3222	0.3176	0.3138	0.3102	0.3074	0.3055	0.3038	0.3012	0.2994	0.2978	0.2964	0.2952
4.00	0.3818	0.3874	0.3862	0.3837	0.3805	0.3774	0.3743	0.3717	0.3699	0.3683	0.3657	0.3639	0.3622	0.3608	0.3596
4.50	0.4355	0.4430	0.4435	0.4431	0.4415	0.4393	0.4369	0.4349	0.4335	0.4320	0.4298	0.4281	0.4265	0.4252	0.4240
5.00	0.4887	0.4975	0.4992	0.5004	0.5006	0.4994	0.4978	0.4966	0.4956	0.4945	0.4928	0.4914	0.4900	0.4888	0.4877
5.50	0.5413	0.5508	0.5535	0.5561	0.5579	0.5577	0.5570	0.5566	0.5561	0.5554	0.5543	0.5534	0.5522	0.5512	0.5503
6.00	0.5936	0.6034	0.6066	0.6103	0.6135	0.6143	0.6144	0.6149	0.6149	0.6147	0.6143	0.6138	0.6129	0.6121	0.6113
6.50	0.6454	0.6553	0.6590	0.6634	0.6676	0.6694	0.6703	0.6715	0.6720	0.6724	0.6726	0.6727	0.6721	0.6715	0.6708
7.00	0.6969	0.7068	0.7105	0.7155	0.7205	0.7230	0.7246	0.7265	0.7276	0.7284	0.7293	0.7299	0.7296	0.7291	0.7286
7.50	0.7482	0.7577	0.7613	0.7666	0.7722	0.7754	0.7776	0.7802	0.7817	0.7829	0.7844	0.7854	0.7855	0.7852	0.7848
8.00	0.7991	0.8082	0.8114	0.8170	0.8230	0.8266	0.8293	0.8324	0.8344	0.8360	0.8391	0.8395	0.8398	0.8397	0.8394
8.50	0.8497	0.8582	0.8611	0.8666	0.8729	0.8768	0.8799	0.8835	0.8858	0.8878	0.8914	0.8920	0.8926	0.8927	0.8925
9.00	0.9000	0.9078	0.9102	0.9157	0.9220	0.9261	0.9295	0.9334	0.9360	0.9382	0.9423	0.9432	0.9440	0.9442	0.9441
9.50	0.9500	0.9570	0.9588	0.9641	0.9704	0.9746	0.9782	0.9824	0.9852	0.9876	0.9920	0.9932	0.9941	0.9944	0.9944
10.00	0.9998	1.0059	1.0071	1.0121	1.0181	1.0223	1.0260	1.0304	1.0334	1.0359	1.0407	1.0420	1.0430	1.0434	1.0435
10.50	1.0492	1.0544	1.0549	1.0595	1.0653	1.0694	1.0731	1.0776	1.0806	1.0833	1.0883	1.0897	1.0908	1.0913	1.0914
11.00	1.0985	1.1026	1.1024	1.1065	1.1119	1.1159	1.1195	1.1239	1.1271	1.1298	1.1349	1.1364	1.1375	1.1380	1.1381
11.50	1.1475	1.1506	1.1496	1.1530	1.1580	1.1618	1.1653	1.1696	1.1728	1.1755	1.1807	1.1822	1.1832	1.1837	1.1839
12.00	1.1963	1.1983	1.1964	1.1992	1.2037	1.2072	1.2105	1.2147	1.2178	1.2205	1.2256	1.2270	1.2281	1.2285	1.2287
12.50	1.2449	1.2458	1.2430	1.2449	1.2490	1.2522	1.2551	1.2592	1.2622	1.2648	1.2698	1.2711	1.2720	1.2724	1.2725
13.00	1.2934	1.2931	1.2893	1.2903	1.2939	1.2967	1.2993	1.3031	1.3060	1.3084	1.3131	1.3143	1.3152	1.3155	1.3156
13.50	1.3417	1.3402	1.3354	1.3354	1.3384	1.3408	1.3430	1.3465	1.3492	1.3514	1.3558	1.3567	1.3575	1.3578	1.3578
14.00	1.3899	1.3870	1.3812	1.3862	1.3825	1.3845	1.3862	1.3894	1.3918	1.3938	1.3977	1.3984	1.3991	1.3993	1.3992
14.50	1.4380	1.4337	1.4268	1.4247	1.4263	1.4278	1.4290	1.4319	1.4339	1.4356	1.4390	1.4395	1.4400	1.4401	1.4400
15.00	1.4860	1.4803	1.4722	1.4689	1.4698	1.4708	1.4714	1.4739	1.4756	1.4769	1.4797	1.4798	1.4802	1.4802	1.4800
15.50	1.5338	1.5266	1.5174	1.5129	1.5130	1.5135	1.5134	1.5155	1.5168	1.5177	1.5198	1.5196	1.5197	1.5197	1.5194
16.00	1.5815	1.5728	1.5625	1.5566	1.5559	1.5558	1.5551	1.5567	1.5575	1.5580	1.5594	1.5587	1.5587	1.5585	1.5582
16.50	1.6291	1.6189	1.6073	1.6001	1.5985	1.5979	1.5964	1.5976	1.5978	1.5979	1.5984	1.5973	1.5971	1.5968	1.5964
17.00	1.6766	1.6649	1.6520	1.6434	1.6409	1.6397	1.6374	1.6381	1.6378	1.6373	1.6370	1.6354	1.6350	1.6346	1.6341
17.50	1.7241	1.7107	1.6966	1.6865	1.6830	1.6812	1.6781	1.6783	1.6773	1.6764	1.6750	1.6730	1.6723	1.6718	1.6712
18.00	1.7714	1.7564	1.7410	1.7293	1.7249	1.7225	1.7186	1.7181	1.7166	1.7150	1.7127	1.7100	1.7091	1.7085	1.7078
18.50	1.8187	1.8020	1.7853	1.7720	1.7666	1.7635	1.7587	1.7577	1.7554	1.7533	1.7499	1.7466	1.7455	1.7447	1.7439
19.00	1.8659	1.8475	1.8294	1.8146	1.8081	1.8043	1.7986	1.7970	1.7940	1.7912	1.7866	1.7828	1.7814	1.7805	1.7796
19.50	1.9130	1.8929	1.8734	1.8569	1.8493	1.8449	1.8382	1.8360	1.8322	1.8288	1.8280	1.8186	1.8169	1.8158	1.8148
20.00	1.9600	1.9382	1.9173	1.8991	1.8904	1.8853	1.8776	1.8747	1.8702	1.8661	1.8590	1.8540	1.8519	1.8508	1.8496
20.50	2.0070	1.9834	1.9611	1.9412	1.9314	1.9255	1.9168	1.9132	1.9079	1.9031	1.8947	1.8889	1.8866	1.8853	1.8840
21.00	2.0539	2.0285	2.0048	1.9831	1.9721	1.9655	1.9557	1.9515	1.9453	1.9397	1.9300	1.9235	1.9209	1.9195	1.9180
21.50	2.1007	2.0736	2.0484	2.0248	2.0127	2.0054	1.9944	1.9895	1.9824	1.9761	1.9650	1.9578	1.9549	1.9532	1.9517
22.00	2.1475	2.1185	2.0918	2.0665	2.0531	2.0450	2.0330	2.0273	2.0193	2.0122	1.9997	1.9917	1.9884	1.9867	1.9850
22.50	2.1943	2.1634	2.1352	2.1080	2.0934	2.0845	2.0713	2.0649	2.0560	2.0481	2.0341	2.0253	2.0217	2.0198	2.0179
23.00	2.2410	2.2082	2.1785	2.1494	2.1335	2.1239	2.1095	2.1024	2.0924	2.0837	2.0681	2.0586	2.0546	2.0525	2.0506
23.50	2.2876	2.2529	2.2217	2.1906	2.1735	2.1631	2.1475	2.1396	2.1286	2.1191	2.1019	2.0916	2.0872	2.0850	2.0829
24.00	2.3342	2.2976	2.2648	2.2318	2.2134	2.2021	2.1853	2.1766	2.1646	2.1542	2.1355	2.1242	2.1196	2.1171	2.1149
24.50	2.3807	2.3422	2.3079	2.2728	2.2531	2.2410	2.2229	2.2135	2.2005	2.1891	2.1687	2.1567	2.1516	2.1490	2.1466
25.00	2.4272	2.3867	2.3509	2.3138	2.2927	2.2798	2.2604	2.2502	2.2361	2.2238	2.2017	2.1888	2.1834	2.1806	2.1780
25.50	2.4736	2.4312	2.3937	2.3546	2.3322	2.3184	2.2978	2.2867	2.2715	2.2583	2.2345	2.2207	2.2149	2.2119	2.2092
26.00	2.5200	2.4756	2.4366	2.3953	2.3716	2.3569	2.3350	2.3230	2.3067	2.2927	2.2671	2.2523	2.2461	2.2430	2.2401
26.50	2.5664	2.5200	2.4793	2.4360	2.4109	2.3953	2.3720	2.3592	2.3418	2.3268	2.2994	2.2837	2.2771	2.2738	2.2707
27.00	2.6127	2.5643	2.5220	2.4766	2.4501	2.4336	2.4089	2.3953	2.3767	2.3607	2.3315	2.3149	2.3078	2.3044	2.3011
27.50	2.6590	2.6086	2.5646	2.5170	2.4891	2.4718	2.4457	2.4312	2.4115	2.3944	2.3634	2.3458	2.3384	2.3347	2.3313
28.00	2.7053	2.6528	2.6072	2.5574	2.5281	2.5098	2.4824	2.4670	2.4460	2.4280	2.3951	2.3765	2.3687	2.3648	2.3612
28.50	2.7515	2.6969	2.6497	2.5977	2.5669	2.5478	2.5189	2.5026	2.4805	2.4614	2.4266	2.4070	2.3987	2.3947	2.3909
29.00	2.7977	2.7410	2.6921	2.6380	2.6057	2.5856	2.5553	2.5382	2.5148	2.4947	2.4579	2.4373	2.4286	2.4244	2.4205
29.50	2.8438	2.7851	2.7345	2.6781	2.6444	2.6234	2.5916	2.5736	2.5489	2.5278	2.4890	2.4674	2.4583	2.4538	2.4497
30.00	2.8899	2.8291	2.7769	2.7182	2.6830	2.6610	2.6278	2.6088	2.5829	2.5607	2.5200	2.4974	2.4878	2.4831	2.4788

TABLE 34.2—EXTENDED SUKKAR-CORNELL INTEGRAL FOR BHP CALCULATION (continued)

$$\int_{0.2}^{p_{pr}} \frac{(z/p_{pr})dp_{pr}}{1+B(z/p_{pr})^2}$$

Pseudoreduced temperature for $B = 20.0$

p_{pr}	1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.8	1.9	2.0	2.2	2.4	2.6	2.8	3.0
0.20	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.50	0.0058	0.0056	0.0055	0.0055	0.0054	0.0054	0.0053	0.0053	0.0053	0.0053	0.0052	0.0052	0.0052	0.0052	0.0052
1.00	0.0294	0.0272	0.0262	0.0255	0.0250	0.0246	0.0243	0.0241	0.0240	0.0239	0.0237	0.0236	0.0235	0.0234	0.0233
1.50	0.0740	0.0649	0.0610	0.0587	0.0572	0.0561	0.0551	0.0545	0.0541	0.0537	0.0532	0.0528	0.0525	0.0522	0.0520
2.00	'0.1295	0.1156	0.1077	0.1030	0.0998	0.0976	0.0958	0.0945	0.0937	0.0930	0.0918	0.0911	0.0905	0.0900	0.0895
2.50	0.1832	0.1712	0.1614	0.1547	0.1498	0.1465	0.1438	0.1417	0.1404	0.1393	0.1376	0.1364	0.1354	0.1346	0.1339
3.00	0.2350	0.2264	0.2172	0.2099	0.2040	0.1999	0.1964	0.1937	0.1920	0.1904	0.1882	0.1867	0.1853	0.1842	0.1832
3.50	0.2860	0.2801	0.2725	0.2657	0.2597	0.2553	0.2514	0.2484	0.2463	0.2445	0.2419	0.2401	0.2384	0.2371	0.2359
4.00	0.3365	0.3326	0.3264	0.3208	0.3154	0.3111	0.3073	0.3041	0.3020	0.3000	0.2972	0.2952	0.2934	0.2919	0.2906
4.50	0.3865	0.3841	0.3790	0.3747	0.3703	0.3664	0.3629	0.3599	0.3578	0.3559	0.3531	0.3510	0.3492	0.3476	0.3462
5.00	0.4360	0.4346	0.4305	0.4273	0.4240	0.4208	0.4177	0.4151	0.4132	0.4114	0.4088	0.4068	0.4050	0.4034	0.4021
5.50	0.4852	0.4843	0.4809	0.4787	0.4765	0.4740	0.4714	0.4594	0.4678	0.4662	0.4639	0.4622	0.4604	0.4589	0.4577
6.00	0.5341	0.5335	0.5305	0.5291	0.5279	0.5261	0.5241	0.5226	0.5213	0.5201	0.5182	0.5167	0.5151	0.5137	0.5125
6.50	0.5827	0.5821	0.5794	0.5786	0.5783	0.5771	0.5756	0.5747	0.5738	0.5729	0.5714	0.5703	0.5689	0.5676	0.5665
7.00	0.6310	0.6304	0.6277	0.6274	0.6276	0.6270	0.6261	0.6257	0.6252	0.6246	0.6236	0.6228	0.6216	0.6205	0.6194
7.50	0.6791	0.6782	0.6755	0.6754	0.6761	0.6760	0.6755	0.6756	0.6754	0.6752	0.6746	0.6741	0.6732	0.6722	0.6712
8.00	0.7269	0.7257	0.7227	0.7228	0.7238	0.7241	0.7240	0.7245	0.7247	0.7247	0.7251	0.7244	0.7237	0.7227	0.7219
8.50	0.7745	0.7728	0.7695	0.7696	0.7708	0.7714	0.7716	0.7725	0.7729	0.7732	0.7740	0.7735	0.7730	0.7227	0.7714
9.00	0.8219	0.8196	0.8159	0.8160	0.8172	0.8179	0.8184	0.8195	0.8202	0.8207	0.8218	0.8216	0.8212	0.8205	0.8198
9.50	0.8690	0.8661	0.8620	0.8618	0.8631	0.8638	0.8644	0.8658	0.8666	0.8673	0.8687	0.8687	0.8684	0.8678	0.8672
10.00	0.9159	0.9123	0.9077	0.9073	0.9083	0.9091	0.9098	0.9113	0.9123	0.9131	0.9147	0.9148	0.9146	0.9141	0.9135
10.50	0.9626	0.9582	0.9530	0.9523	0.9531	0.9538	0.9545	0.9561	0.9571	0.9580	0.9599	0.9601	0.9599	0.9595	0.9589
11.00	1.0091	1.0039	0.9981	0.9969	0.9975	0.9980	0.9987	1.0002	1.0014	1.0023	1.0043	1.0045	1.0043	1.0039	1.0034
11.50	1.0554	1.0494	1.0429	1.0412	1.0414	1.0418	1.0423	1.0438	1.0450	1.0459	1.0479	1.0481	1.0479	1.0475	1.0470
12.00	1.1016	1.0946	1.0874	0.0851	1.0849	1.0851	1.0855	1.0868	1.0879	1.0888	1.0908	1.0909	1.0908	1.0903	1.0898
12.50	1.1476	1.1397	1.1317	1.1288	1.1282	1.1280	1.1282	1.1294	1.1304	1.1312	1.1331	1.1331	1.1328	1.1323	1.1318
13.00	1.1935	1.1846	1.1758	1.1721	1.1710	1.1706	1.1704	1.1714	1.1723	1.1730	1.1746	1.1745	1.1742	1.1736	1.1731
13.50	1.2392	1.2293	1.2197	1.2151	1.2136	1.2128	1.2122	1.2130	1.2137	1.2142	1.2156	1.2153	1.2149	1.2143	1.2136
14.00	1.2849	1.2739	1.2633	1.2579	1.2558	1.2547	1.2537	1.2542	1.2547	1.2549	1.2559	1.2554	1.2549	1.2542	1.2535
14.50	1.3304	1.3183	1.3068	1.3005	1.2977	1.2962	1.2948	1.2949	1.2952	1.2952	1.2957	1.2949	1.2943	1.2935	1.2928
15.00	1.3759	1.3625	1.3501	1.3428	1.3394	1.3375	1.3355	1.3353	1.3352	1.3349	1.3349	1.3339	1.3331	1.3322	1.3315
15.50	1.4212	1.4067	1.3933	1.3849	1.3808	1.3784	1.3759	1.3754	1.3749	1.3743	1.3736	1.3723	1.3713	1.3704	1.3695
16.00	1.4665	1.4507	1.4363	1.4267	1.4220	1.4191	1.4150	1.4151	1.4142	1.4132	1.4118	1.4101	1.4090	1.4080	1.4071
16.50	1.5116	1.4945	1.4792	1.4684	1.4629	1.4595	1.4558	1.4544	1.4531	1.4517	1.4496	1.4475	1.4462	1.4451	1.4441
17.00	1.5567	1.5383	1.5219	1.5099	1.5036	1.4997	1.4953	1.4935	1.4916	1.4898	1.4869	1.4844	1.4829	1.4817	1.4806
17.50	1.6017	1.5820	1.5645	1.5512	1.5441	1.5397	1.5345	1.5323	1.5298	1.5275	1.5238	1.5208	1.5191	1.5178	1.5166
18.00	1.6467	1.6256	1.6069	1.5924	1.5844	1.5794	1.5735	1.5708	1.5678	1.5649	1.5603	1.5568	1.5549	1.5534	1.5522
18.50	1.6916	1.6691	1.6493	1.6334	1.6245	1.6190	1.6123	1.6090	1.6054	1.6020	1.5964	1.5924	1.5902	1.5887	1.5873
19.00	1.7364	1.7125	1.6915	1.6742	1.6644	1.6583	1.6508	1.6470	1.6427	1.6388	1.6321	1.6275	1.6252	1.6235	1.6220
19.50	1.7811	1.7558	1.7336	1.7149	1.7042	1.6975	1.6891	1.6847	1.6797	1.6752	1.6675	1.6623	1.6597	1.6579	1.6563
20.00	1.8258	1.7990	1.7757	1.7555	1.7438	1.7364	1.7271	1.7222	1.7165	1.7114	1.7025	1.6967	1.6938	1.6919	1.6902
20.50	1.8705	1.8421	1.8176	1.7959	1.7832	1.7752	1.7650	1.7595	1.7530	1.7473	1.7372	1.7308	1.7276	1.7256	1.7238
21.00	1.9150	1.8852	1.8594	1.8362	1.8225	1.8139	1.8027	1.7965	1.7893	1.7829	1.7716	1.7645	1.7611	1.7589	1.7570
21.50	1.9596	1.9282	1.9012	1.8763	1.8616	1.8523	1.8401	1.8334	1.8254	1.8183	1.8056	1.7979	1.7942	1.7918	1.7898
22.00	2.0041	1.9711	1.9429	1.9164	1.9006	1.8906	1.8774	1.8700	1.8612	1.8534	1.8394	1.8310	1.8270	1.8245	1.8223
22.50	2.0485	2.0140	1.9844	1.9563	1.9395	1.9288	1.9146	1.9065	1.8968	1.8882	1.8730	1.8638	1.8595	1.8568	1.8545
23.00	2.0929	2.0568	2.0259	1.9962	1.9782	1.9668	1.9516	1.9428	1.9322	1.9229	1.9062	1.8963	1.8916	1.8889	1.8864
23.50	2.1372	2.0995	2.0674	2.0359	2.0168	2.0047	1.9684	1.9789	1.9674	1.9573	1.9392	1.9286	1.9235	1.9206	1.9180
24.00	2.1815	2.1422	2.1087	2.0756	2.0553	2.0425	2.0250	2.0149	2.0025	1.9916	1.9719	1.9605	1.9551	1.9521	1.9493
24.50	2.2258	2.1849	2.1500	2.1151	2.0937	2.0801	2.0615	2.0507	2.0373	2.0256	2.0044	1.9922	1.9865	1.9832	1.9804
25.00	2.2700	2.2274	2.1912	2.1546	2.1319	2.1176	2.0979	2.0863	2.0719	2.0594	2.0367	2.0237	2.0176	2.0142	2.0112
25.50	2.3142	2.2700	2.2324	2.1939	2.1701	2.1550	2.1341	2.1218	2.1064	2.0930	2.0687	2.0549	2.0484	2.0449	2.0417
26.00	2.3584	2.3124	2.2735	2.2332	2.2082	2.1923	2.1702	2.1571	2.1408	2.1265	2.1005	2.0858	2.0790	2.0753	2.0720
26.50	2.4025	2.3549	2.3145	2.2724	2.2461	2.2295	2.2062	2.1923	2.1749	2.1598	2.1321	2.1166	2.1094	2.1055	2.1020
27.00	2.4466	2.3973	2.3565	2.3115	2.2840	2.2665	2.2420	2.2274	2.2089	2.1929	2.1636	2.1471	2.1395	2.1355	2.1318
27.50	2.4907	2.4396	2.3964	2.3505	2.3218	2.3035	2.2778	2.2623	2.2428	2.2258	2.1948	2.1774	2.1695	2.1652	2.1614
28.00	2.5347	2.4819	2.4373	2.3895	2.3595	2.3404	2.3134	2.2971	2.2765	2.2586	2.2258	2.2075	2.1992	2.1948	2.1908
28.50	2.5707	2.5243	2.4781	2.4204	2.3971	2.3772	2.3409	2.3110	2.3100	2.2912	2.2566	2.2375	2.2287	2.2241	2.2200
29.00	2.6228	2.5664	2.5189	2.4672	2.4146	2.4119	2.3848	2.3664	2.3435	2.3217	2.2873	2.2675	2.2580	2.2552	2.2600
29.50	2.6666	2.6085	2.5596	2.5060	2.4720	2.4504	2.4195	2.4008	2.3768	2.3560	2.3178	2.2967	2.2871	2.2822	2.2777
30.00	2.7106	2.6507	2.6003	2.5447	2.5094	2.4870	2.4547	2.4352	2.4100	2.3882	2.3481	2.3261	2.3161	2.3109	2.3063

TABLE 34.2—EXTENDED SUKKAR-CORNELL INTEGRAL FOR BHP CALCULATION (continued)

$$\int_{0.2}^{p_{pr}} \frac{(z/p_{pr})dp_{pr}}{1+B(z/p_{pr})^2}$$

Pseudoreduced temperature for $B = 25.0$

p_{pr}	1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.8	1.9	2.0	2.2	2.4	2.6	2.8	3.0
0.20	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.50	0.0047	0.0045	0.0044	0.0044	0.0043	0.0043	0.0043	0.0042	0.0042	0.0042	0.0042	0.0042	0.0042	0.0042	0.0042
1.00	0.0237	0.0219	0.0211	0.0205	0.0201	0.0198	0.0196	0.0194	0.0193	0.0192	0.0191	0.0187	0.0189	0.0188	0.0187
1.50	0.0611	0.0529	0.0496	0.0477	0.0464	0.0454	0.0446	0.0441	0.0438	0.0435	0.0430	0.0427	0.0424	0.0422	0.0420
2.00	0.1106	0.0961	0.0888	0.0846	0.0818	0.0798	0.0783	0.0771	0.0764	0.0758	0.0749	0.0742	0.0737	0.0733	0.0729
2.50	0.1598	0.1453	0.1352	0.1287	0.1241	0.1211	0.1186	0.1168	0.1156	0.1146	0.1131	0.1121	0.1111	0.1104	0.1098
3.00	0.2079	0.1952	0.1846	0.1769	0.1711	0.1670	0.1637	0.1612	0.1596	0.1581	0.1561	0.1547	0.1534	0.1524	0.1515
3.50	0.2554	0.2444	0.2346	0.2267	0.2202	0.2156	0.2117	0.2087	0.2067	0.2049	0.2024	0.2007	0.1991	0.1978	0.1967
4.00	0.3025	0.2930	0.2840	0.2766	0.2702	0.2654	0.2613	0.2579	0.2557	0.2537	0.2508	0.2488	0.2470	0.2455	0.2442
4.50	0.3492	0.3408	0.3325	0.3260	0.3200	0.3154	0.3112	0.3078	0.3055	0.3034	0.3004	0.2982	0.2962	0.2946	0.2932
5.00	0.3957	0.3879	0.3803	0.3745	0.3693	0.3650	0.3610	0.3578	0.3555	0.3534	0.3503	0.3481	0.3461	0.3444	0.3429
5.50	0.4418	0.4345	0.4274	0.4223	0.4178	0.4139	0.4103	0.4073	0.4052	0.4031	0.4002	0.3980	0.3961	0.3963	0.3929
6.00	0.4878	0.4806	0.4739	0.4694	0.4656	0.4622	0.4589	0.4563	0.4543	0.4525	0.4498	0.4477	0.4458	0.4441	0.4428
6.50	0.5335	0.5263	0.5198	0.5158	0.5126	0.5097	0.5068	0.5045	0.5028	0.5012	0.4988	0.4969	0.4951	0.4935	0.4922
7.00	0.5790	0.5718	0.5653	0.5616	0.5589	0.5564	0.5539	0.5520	0.5506	0.5492	0.5471	0.5454	0.5437	0.5422	0.5409
7.50	0.6243	0.6169	0.6104	0.6069	0.6045	0.6024	0.6003	0.5987	0.5975	0.5964	0.5946	0.5932	0.5917	0.5902	0.5890
8.00	0.6694	0.6618	0.6550	0.6516	0.6495	0.6477	0.6459	0.6447	0.6437	0.6428	0.6415	0.6401	0.6388	0.6374	0.6362
8.50	0.7143	0.7063	0.6993	0.6960	0.6940	0.6924	0.6908	0.6899	0.6892	0.6884	0.6874	0.6862	0.6850	0.6837	0.6826
9.00	0.7591	0.7506	0.7433	'0.7399	0.7380	0.7365	0.7351	0.7344	0.7338	0.7333	0.7325	0.7315	0.7304	0.7292	0.7282
9.50	0.8036	0.7946	0.7870	0.7834	0.7814	0.7800	0.7788	0.7783	0.7778	0.7774	0.7769	0.7760	0.7750	0.7739	0.7730
10.00	0.8480	0.8384	0.8303	0.8266	0.8245	0.8231	0.8219	0.8215	0.8212	0.8208	0.8205	0.8183	0.8189	0.8178	0.8169
10.50	0.8922	0.8820	0.8735	0.8695	0.8671	0.8657	0.8645	0.8641	0.8639	0.8636	0.8635	0.8628	0.8619	0.8609	0.8600
11.00	0.9362	0.9254	0.9163	0.9120	0.9094	0.9078	0.9056	0.9063	0.9061	0.9058	0.9058	0.9052	0.9043	0.9033	0.9024
11.50	0.9801	0.9686	0.9590	0.9542	0.9514	0.9496	0.9483	0.9479	0.9477	0.9475	0.9475	0.9468	0.9459	0.9449	0.9440
12.00	1.0239	1.0117	1.0014	0.9961	0.9930	0.9910	0.9896	0.9891	0.9889	0.9886	0.9885	0.9879	0.9869	0.9859	0.9850
12.50	1.0676	1.0545	1.0437	1.0378	1.0343	1.0321	1.0304	1.0298	1.0295	1.0292	1.0290	1..0283	1.0273	1.0262	1.0253
13.00	1.1111	1.0973	1.0857	1.0792	1.0753	1.0729	1.0709	1.0701	1.0698	1.0693	1.0689	1.0681	1.0670	1.0659	1.0650
13.50	1.1547	1.1398	1.1276	1.1204	1.1161	1.1134	1.1111	1.1101	1.1095	1.1089	1.1083	1.1073	1.1062	1.1050	1.1040
14.00	1.1979	1.1823	1.1693	1.1614	1.1566	1.1535	1.1509	1.1496	1.1489	1.1481	1.1472	1.1459	1.1447	1.1435	1.1425
14.50	1.2412	1.2246	1.2109	1.2021	1.1968	1.1934	1.1904	1.1889	1.1879	1.1868	1.1855	1.1840	1.1827	1.1815	1.1804
15.00	1.2844	1.2668	1.2523	1.2427	1.2368	1.2331	1.2296	1.2278	1.2265	1.2252	1.2234	1.2217	1.2202	1.2189	1.2177
15.50	1.3275	1.3089	1.2936	1.2830	1.2766	1.2725	1.2685	1.2663	1.2647	1.2631	1.2608	1.2588	1.2572	1.2558	1.2546
16.00	1.3705	1.3509	1.3347	1.3232	1.3161	2.3116	1.3071	1.3046	1.3026	1.3007	1.2978	1.2954	1.2937	1.2922	1.2909
16.50	1.4135	1.3928	1.3757	1.3632	1.3555	1.3505	1.3455	1.3426	1.3402	1.3379	1.3343	1.3316	1.3298	1.3291	1.3268
17.00	1.4564	1.4346	1.4166	1.4031	1.3947	1.3892	1.3836	1.3803	1.3775	1.3748	1.3705	1.3674	1.3653	1.3637	1.3623
17.50	1.4992	1.4763	1.4574	1.4428	1.4336	1.4278	1.4215	1.4178	1.4145	1.4114	1.4062	1.4028	1.4005	1.3987	1.3973
18.00	1.5420	1.5180	1.4981	1.4823	1.4724	1.4661	1.4591	1.4550	1.4512	1.4476	1.4417	1.4377	1.4353	1.4334	1.4318
18.50	1.5847	1.5595	1.5387	1.5217	1.5111	1.5042	1.4965	1.4920	1.4876	1.4835	1.4767	1.4723	1.4697	1.4677	1.4660
19.00	1.6274	1.6010	1.5792	1.5610	1.5496	1.5422	1.5338	1.5287	1.5238	1.5192	1.5114	1.5065	1.5036	1.5015	1.4998
19.50	1.6700	1.6424	1.6196	1.6002	1.5879	1.5800	1.5708	1.5653	1.5597	1.5546	1.5458	1.5404	1.5373	1.5351	1.5332
20.00	1.7126	1.6837	1.6599	1.6392	1.6261	1.6176	1.6076	1.6016	1.5954	1.5897	1.5799	1.5739	1.5706	1.5692	1.5663
20.50	1.7551	1.7250	1.7001	1.6781	1.6641	1.6551	1.6443	1.6377	1.6308	1.6246	1.6137	1.6071	1.6035	1.6011	1.5990
21.00	1.7975	1.7662	1.7403	1.7169	1.7020	1.6924	1.6808	1.6736	1.6660	1.6592	1.6472	1.6400	1.5362	1.6336	1.6314
21.50	1.8400	1.8073	1.7803	1.7556	1.7398	1.7296	1.7171	1.7094	1.7011	1.6936	1.6804	1.6726	1.5685	1.6658	1.6635
22.00	1.8824	1.8484	1.8203	1.7942	1.7775	1.7667	1.7532	1.7450	1.7359	1.7278	1.7134	1.7049	1.7005	1.6977	1.6953
22.50	1.9247	1.8895	1.8603	1.8327	1.8150	1.8036	1.7892	1.7804	1.7705	1.7617	1.7460	1.7370	1.7322	1.7293	1.7267
23.00	1.9670	1.9304	1.9001	1.8711	1.8524	1.8404	1.8251	1.8156	1.8049	1.7955	1.7785	1.7687	1.7637	1.7606	1.7579
23.50	2.0093	1.9714	1.9399	1.9094	1.8898	1.8771	1.8608	1.8507	1.8392	1.8290	1.8107	1.8002	1.7949	1.7916	1.7889
24.00	2.0516	2.0122	1.9797	1.9477	1.9270	1.9136	1.8964	1.8856	1.8733	1.8623	1.8427	1.8315	1.8258	1.8224	1.8195
24.50	2.0938	2.0531	2.0193	1.9858	1.9641	1.9501	1.9318	1.9204	1.9072	1.8955	1.8744	1.8625	1.8565	1.8530	1.8499
25.00	2.1360	2.0938	2.0590	2.0239	2.0011	1.9864	1.9671	1.9550	1.9409	1.9285	1.9060	1.8933	1.8870	1.8833	1.8801
25.50	2.1761	2.1346	2.0985	2.0618	2.0380	2.0226	2.0023	1.9895	1.9745	1.9613	1.9373	1.9238	1.9172	1.9133	1.9100
26.00	2.2202	2.1753	2.1380	2.0998	2.0749	2.0588	2.0373	2.0239	2.0079	1.9939	1.9684	1.9542	1.9472	1.9431	1.9397
26.50	2.2623	2.2159	2.1775	2.1376	2.1116	2.0948	2.0723	2.0581	2.0412	2.0264	1.9994	1.9843	1.9769	1.9728	1.9692
27.00	2.3044	2.2566	2.2169	2.1754	2.1483	2.1307	2.1071	2.0923	2.0744	2.0587	2.0301	2.0142	2.0065	2.0022	1.9984
27.50	2.3464	2.2971	2..2562	2.2131	2.1848	2.1666	2.1418	2.1263	2.1074	2.0909	2.0607	2.0440	2.0359	2.0314	2.0275
28.00	2.3885	2.3377	2.2955	2.2507	2.2213	2.2024	2.1764	2.1601	2.1403	2.1229	2.0911	2.0735	2.0650	2.0603	2.0563
28.50	2.4305	2.3782	2.3348	2.2883	2.2578	2.2380	2.2110	2.1939	2.1730	2.1548	2.1213	2.1028	2.0940	2.0891	2.0849
29.00	2.4724	2.4186	2.3740	2.3258	2.2941	2.2736	2.2454	2.2276	2.2056	2.1865	2.1513	2.1320	2.1228	2.1178	2.1134
29.50	2.5144	2.4591	2.4132	2.3632	2.3304	2.3091	2.2797	2.2611	2.2381	2.2181	2.1812	2.1610	2.1514	2.1462	2.1417
30.00	2.5563	2.4995	2.4523	2.4006	2.3666	2.3446	2.3139	2.2946	2.2705	2.2496	2.2110	2.1898	2.1798	2.1744	2.1698

TABLE 34.2—EXTENDED SUKKAR-CORNELL INTEGRAL FOR BHP CALCULATION (continued)

$$\int_{0.2}^{p_{pr}} \frac{(z/p_{pr})dp_{pr}}{1+B(z/p_{pr})^2}$$

Pseudoreduced temperature for $B = 30.0$

p_{pr}	1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.8	1.9	2.0	2.2	2.4	2.6	2.8	3.0
0.20	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.50	0.0039	0.0038	0.0037	0.0037	0.0036	0.0036	0.0036	0.0035	0.0035	0.0035	0.0035	0.0035	0.0035	0.0035	0.0035
1.00	0.0199	0.0184	0.0176	0.0172	0.0168	0.0166	0.0164	0.0162	0.0162	0.0161	0.0159	0.0158	0.0158	0.0157	0.0157
1.50	0.0521	0.0447	0.0418	0.0401	0.0390	0.0382	0.0375	0.0371	0.0368	0.0365	0.0361	0.0358	0.0356	0.0355	0.0353
2.00	0.0967	0.0823	0.0755	0.0718	0.0692	0.0676	0.0662	0.0652	0.0646	0.0640	0.0632	0.0626	0.0621	0.0618	0.0615
2.50	0.1422	0.1264	0.1164	0.1103	0.1060	0.1033	0.1010	0.0993	0.0963	0.0974	0.0960	0.0951	0.0943	0.0937	0.0931
3.00	0.1870	0.1719	0.1608	0.1531	0.1474	0.1436	0.1404	0.1381	0.1366	0.1353	0.1334	0.1321	0.1309	0.1300	0.1292
3.50	0.2314	0.2174	0.2063	0.1980	0.1914	0.1869	0.1831	0.1801	0.1782	0.1765	0.1741	0.1725	0.1710	0.1697	0.1687
4.00	0.2756	0.2625	0.2519	0.2436	0.2367	0.2318	0.2275	0.2242	0.2219	0.2199	0.2172	0.2152	0.2135	0.2120	0.2108
4.50	0.3195	0.3071	0.2970	0.2891	0.2823	0.2778	0.2729	0.2693	0.2669	0.2647	0.2617	0.2594	0.2575	0.2559	0.2545
5.00	0.3632	0.3513	0.3416	0.3343	0.3278	0.3229	0.3186	0.3149	0.3124	0.3101	0.3069	0.3046	0.3025	0.3008	0.2993
5.50	0.4067	0.3951	0.3858	0.3789	0.3729	0.3683	0.3641	0.3605	0.3580	0.3558	0.3525	0.3501	0.3480	0.3462	0.3448
6.00	0.4500	0.4386	0.4295	0.4230	0.4175	0.4132	0.4092	0.4059	0.4035	0.4013	0.3981	0.3957	0.3937	0.3919	0.3904
6.50	0.4931	0.4817	0.4728	0.4667	0.4616	0.4576	0.4539	0.4508	0.4486	0.4465	0.4435	0.4412	0.4392	0.4374	0.4359
7.00	0.5361	0.5247	0.5158	0.5099	0.5052	0.5015	0.4981	0.4952	0.4932	0.4913	0.4884	0.4863	0.4843	0.4826	0.4812
7.50	0.5789	0.5674	0.5584	0.5527	0.5483	0.5449	0.5417	0.5391	0.5372	0.5355	0.5329	0.5309	0.5291	0.5274	0.5260
8.00	0.6216	0.6098	0.6007	0.5951	0.5909	0.5877	0.5848	0.5824	0.5808	0.5792	0.5767	0.5749	0.5732	0.5716	0.5703
8.50	0.6642	0.6521	0.6428	0.6372	0.6331	0.6301	0.6273	0.6252	0.6237	0.6223	0.6200	0.6184	0.6168	0.6152	0.6139
9.00	0.7066	0.6941	0.6846	0.6789	0.6749	0.6719	0.6693	0.6674	0.6660	0.6647	0.6627	0.6612	0.6597	0.6582	0.6570
9.50	0.7488	0.7360	0.7261	0.7204	0.7163	0.7134	0.7109	0.7091	0.7078	0.7066	0.7048	0.7034	0.7020	0.7006	0.6994
10.00	0.7909	0.7776	0.7674	0.7615	0.7573	0.7544	0.7520	0.7503	0.7491	0.7480	0.7463	0.7451	0.7436	0.7423	0.7411
10.50	0.8329	0.8191	0.8085	0.8024	0.7980	0.7951	0.7926	0.7910	0.7899	0.7888	0.7873	0.7861	0.7847	0.7833	0.7822
11.00	0.8747	0.8604	0.8494	0.8430	0.8384	0.8354	0.8329	0.8313	0.8302	0.8292	0.8277	0.8265	0.8251	0.8238	0.8227
11.50	0.9165	0.9016	0.8901	0.8833	0.8785	0.8754	0.8728	0.8711	0.8700	0.8690	0.8676	0.8664	0.8650	0.8637	0.8626
12.00	0.9581	0.9426	0.9306	0.9234	0.9183	0.9150	0.9123	0.9106	0.9095	0.9084	0.9070	0.9057	0.9043	0.9030	0.9019
12.50	0.9996	0.9835	0.9710	0.9633	0.9579	0.9544	0.9515	0.9497	0.9485	0.9474	0.9459	0.9446	0.9431	0.9417	0.9406
13.00	1.0411	1.0242	1.0112	1.0030	0.9973	0.9936	0.9904	0.9884	0.9872	0.9860	0.9842	0.9828	0.9813	0.9799	0.9787
13.50	1.0824	1.0649	1.0513	1.0425	1.0364	1.0324	1.0290	1.0268	1.0254	1.0241	1.0222	1.0206	1.0191	1.0176	1.0164
14.00	1.1237	1.1054	1.0912	1.0318	1.0753	1.0710	1.0673	1.0649	1.0634	1.0618	1.0596	1.0579	1.0563	1.0547	1.0535
14.50	1.1649	1.1459	1.1310	1.1209	1.1139	1.1094	1.1054	1.1027	1.1009	1.0992	1.0966	1.0947	1.0930	1.0914	1.0901
15.00	1.2060	1.1862	1.1707	1.1598	1.1524	1.1475	1.1431	1.1402	1.1382	1.1362	1.1332	1.1311	1.1293	1.1276	1.1263
15.50	1.2471	1.2264	1.2102	1.1986	1.1907	1.1855	1.1806	1.1774	1.1751	1.1729	1.1694	1.1670	1.1651	1.1633	1.1620
16.00	1.2681	1.2666	1.2497	1.2372	1.2287	1.2232	1.2179	1.2144	1.2117	1.2092	1.2052	1.2026	1.2005	1.1987	1.1972
16.50	1.3291'	1.3067	1.2890	1.2757	1.2666	1.2607	1.2549	1.2511	1.2481	1.2453	1.2407	1.2377	1.2354	1.2335	1.2320
17.00	1.3700	1.3467	1.3282	1.3140	1.3044	1.2981	1.2917	1.2876	1.2842	1.2810	1.2757	1.2724	1.2700	1.2680	1.2665
17.50	1.4109	1.3866	1.3674	1.3522	1.3419	1.3352	1.3283	1.3238	1.3200	1.3164	1.3105	1.3067	1.3042	1.3021	1.3005
18.00	1.4517	1.4264	1.4064	1.3903	1.3794	1.3722	1.3647	1.3598	1.3555	1.3515	1.3449	1.3407	1.3380	1.3358	1.3341
18.50	1.4924	1.4662	1.4454	1.4282	1.4167	1.4091	1.4009	1.3956	1.3908	1.3864	1.3789	1.3744	1.3714	1.3692	1.3674
19.00	1.5332	1.5059	1.4843	1.4661	1.4538	1.4457	1.4370	1.4312	1.4529	1.4211	1.4127	1.4077	1.4045	1.4022	1.4003
19.50	1.5738	1.5456	1.5231	1.5038	1.4908	1.4823	1.4728	1.4666	1.4608	1.4554	1.4462	1.4407	1.4373	1.4349	1.4329
20.00	1.6145	1.5852	1.5618	1.5414	1.5277	1.5187	1.5085	1.5019	1.4954	1.4896	1.4794	1.4734	1.4698	1.4672	1.4652
20.50	1.6551	1.6247	1.6005	1.5789	1.5644	1.5549	1.5440	1.5369	1.5298	1.5235	1.5123	1.5058	1.5019	1.4993	1.4971
21.00	1.6956	1.6642	1.6391	1.6163	1.6011	1.5910	1.5794	1.5718	1.5641	1.5572	1.5449	1.5379	1.5338	1.5310	1.5288
21.50	1.7361	1.7037	1.6776	1.6537	1.6376	1.6270	1.6146	1.6065	1.5981	1.5906	1.5773	1.5697	1.5654	1.5625	1.5601
22.00	1.7766	1.7431	1.7160	1.6909	1.6740	1.6629	1.6497	1.6410	1.6320	1.6239	1.6095	1.6013	1.5967	1.5937	1.5912
22.50	1.8171	1.7824	1.7544	1.7281	1.7103	1.6987	1.6846	1.6754	1.6657	1.6570	1.6414	1.6326	1.6277	1.6246	1.6220
23.00	1.8575	1.8217	1.7928	1.7651	1.7465	1.7343	1.7194	1.7096	1.6992	1.6899	1.6731	1.6636	1.6585	1.6552	1.6525
23.50	1.8979	1.8610	1.8311	1.8021	1.7826	1.7698	1.7541	1.7437	1.7325	1.7226	1.7046	1.6945	1.6890	1.6856	1.6828
24.00	1.9383	1.9002	1.8693	1.8390	1.8186	1.8053	1.7886	1.7777	1.7657	1.7551	1.7358	1.7250	1.7193	1.7158	1.7128
24.50	1.9786	1.9393	1.9075	1.8759	1.8546	1.8406	1.8230	1.8115	1.7987	1.7874	1.7669	1.7554	1.7494	1.7457	1.7426
25.00	2.0189	1.9785	1.9456	1.9127	1.8904	1.8758	1.8573	1.8452	1.8316	1.8196	1.7977	1.7855	1.7792	1.7754	1.7722
25.50	2.0592	2.0176	1.9837	1.9493	1.9262	1.9110	1.8915	1.8788	1.8644	1.8516	1.8284	1.8155	1.8088	1.8048	1.8015
26.00	2.0995	2.0566	2.0217	1.9860	1.9618	1.9460	1.9256	1.9123	1.8970	1.8835	1.8589	1.8452	1.8382	1.8341	1.8306
26.50	2.1397	2.0957	2.0597	2.0226	1.9974	1.9810	1.9596	1.9456	1.9294	1.9152	1.8891	1.8747	1.8674	1.8631	1.8595
27.00	2.1799	2.1346	2.0976	2.0591	2.0330	2.0159	1.9934	1.9788	1.9618	1.9468	1.9192	1.9040	1.8964	1.8920	1.8882
27.50	2.2201	2.1736	2.1355	2.0955	2.0684	2.0507	2.0272	2.0119	1.9940	1.9782	1.9492	1.9332	1.9252	1.9206	1.9167
28.00	2.2603	2.2125	2.1734	2.1319	2.1038	2.0854	2.0609	2.0449	2.0261	2.0095	1.9790	1.9622	1.9538	1.9491	1.9451
28.50	2.3005	2.2514	2.2112	2.1682	2.1391	2.1200	2.0945	2.0779	2.0580	2.0407	2.0086	1.9910	1.9823	1.9774	1.9732
29.00	2.3406	2.2903	2.2490	2.2045	2.1743	2.1546	2.1280	2.1107	2.0899	2.0717	2.0380	2.0196	2.0105	2.0055	2.0012
29.50	2.3807	2.3291	2.2868	2.2407	2.2095	2.1891	2.1614	2.1434	2.1216	2.1026	2.0673	2.0481	2.0386	2.0334	2.0289
30.00	2.4208	2.3679	2.3245	2.2769	2.2446	2.2235	2.1947	2.1760	2.1533	2.1334	2.0965	2.0764	2.0666	2.0612	2.0566

TABLE 34.2—EXTENDED SUKKAR-CORNELL INTEGRAL FOR BHP CALCULATION (continued)

$$\int_{0.2}^{p_{pr}} \frac{(z/p_{pr})dp_{pr}}{1+B(z/p_{pr})^2}$$

Pseudoreduced temperature for $B=35.0$

p_{pr}	1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.8	1.9	2.0	2.2	2.4	2.6	2.8	3.0
0.20	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.50	0.0033	0.0032	0.0032	0.0031	0.0031	0.0031	0.0031	0.0030	0.0030	0.0030	0.0030	0.0030	0.0030	0.0030	0.0030
1.00	0.0171	0.0158	0.0152	0.0148	0.0145	0.0143	0.0141	0.0139	0.0139	0.0138	0.0137	0.0136	0.0136	0.0135	0.0135
1.50	0.0454	0.0387	0.0361	0.0346	0.0336	0.0329	0.0323	0.0320	0.0317	0.0315	0.0311	0.0309	0.0307	0.0305	0.0304
2.00	0.0861	0.0720	0.0657	0.0623	0.0601	0.0585	0.0573	0.0564	0.0559	0.0554	0.0546	0.0542	0.0537	0.0534	0.0531
2.50	0.1283	0.1119	0.1022	0.0965	0.0925	0.0900	0.0879	0.0864	0.0855	0.0847	0.0834	0.0826	0.0819	0.0813	0.0808
3.00	0.1703	0.1538	0.1425	0.1350	0.1295	0.1259	0.1230	0.1208	0.1194	0.1182	0.1165	0.1153	0.1142	0.1134	0.1127
3.50	0.2120	0.1960	0.1844	0.1759	0.1694	0.1650	0.1613	0.1585	0.1567	0.1550	0.1528	0.1513	0.1499	0.1487	0.1478
4.00	0.2536	0.2382	0.2266	0.2179	0.2108	0.2059	0.2017	0.1984	0.1962	0.1942	0.1916	0.1897	0.1880	0.1866	0.1855
4.50	0.2950	0.2800	0.2688	0.2601	0.2529	0.2477	0.2433	0.2396	0.2372	0.2350	0.2320	0.2298	0.2279	0.2263	0.2250
5.00	0.3362	0.3216	0.3106	0.3023	0.2951	0.2899	0.2854	0.2816	0.2790	0.2766	0.2734	0.2710	0.2690	0.2672	0.2658
5.50	0.3773	0.3630	0.3522	0.3442	0.3373	0.3321	0.3276	0.3238	0.3211	0.3187	0.3153	0.3128	0.3107	0.3089	0.3074
6.00	0.4183	0.4040	0.3934	0.3857	0.3791	0.3742	0.3698	0.3660	0.3634	0.3610	0.3576	0.3550	0.3529	0.3510	0.3495
6.50	0.4591	0.4449	0.4344	0.4270	0.4207	0.4159	0.4117	0.4080	0.4055	0.4032	0.3998	0.3972	0.3951	0.3932	0.3918
7.00	0.4999	0.4856	0.4752	0.4679	0.4618	0.4573	0.4532	0.4498	0.4473	0.4451	0.4418	0.4394	0.4373	0.4354	0.4339
7.50	0.5405	0.5261	0.5156	0.5085	0.5026	0.4983	0.4944	0.4912	0.4889	0.4867	0.4836	0.4812	0.4792	0.4774	0.4759
8.00	0.5810	0.5665	0.5558	0.5487	0.5431	0.5390	0.5352	0.5322	0.5300	0.5280	0.5247	0.5227	0.5208	0.5190	0.5175
8.50	0.6214	0.6066	0.5959	0.5888	0.5832	0.5792	0.5756	0.5727	0.5707	0.5688	0.5657	0.5638	0.5619	0.5602	0.5588
9.00	0.6617	0.6466	0.6357	0.6285	0.6230	0.6191	0.6156	0.6129	0.6109	0.6091	0.6062	0.6044	0.6026	0.6009	0.5996
9.50	0.7018	0.6865	0.6753	0.6681	0.6625	0.6586	0.6552	0.6526	0.6507	0.6490	0.6462	0.6445	0.6428	0.6412	0.6398
10.00	0.7419	0.7262	0.7147	0.7073	0.7017	0.6978	0.6945	0.6919	0.6901	0.6885	0.6858	0.6842	0.6825	0.6809	0.6796
10.50	0.7818	0.7657	0.7539	0.7464	0.7406	0.7367	0.7334	0.7308	0.7291	0.7275	0.7250	0.7234	0.7217	0.7201	0.7189
11.00	0.8217	0.8051	0.7930	0.7852	0.7793	0.7753	0.7719	0.7694	0.7677	0.7661	0.7637	0.7621	0.7604	0.7589	0.7576
11.50	0.8614	0.8444	0.8319	0.8239	0.8177	0.8136	0.8102	0.8076	0.8059	0.8043	0.8019	0.8004	0.7987	0.7971	0.7958
12.00	0.9011	0.8836	0.8707	0.8623	0.8559	0.8517	0.8481	0.8455	0.8438	0.8422	0.8398	0.8381	0.8364	0.8349	0.8336
12.50	0.9407	0.9227	0.9094	0.9006	0.8939	0.8895	0.8858	0.8831	0.8813	0.8797	0.8771	0.8755	0.8737	0.8721	0.8708
13.00	0.9803	0.9617	0.9479	0.9386	0.9317	0.9271	0.9232	0.9204	0.9185	0.9168	0.9141	0.9124	0.9106	0.9089	0.9076
13.50	1.0197	1.0006	0.9863	0.9765	0.9693	0.9645	0.9604	0.9574	0.9554	0.9535	0.9507	0.9483	0.9470	0.9453	0.9439
14.00	1.0591	1.0394	1.0246	1.0143	1.0067	1.0017	0.9973	0.9941	0.9920	0.9900	0.9869	0.9848	0.9829	0.9812	0.9798
14.50	1.0985	1.0781	1.0627	1.0519	1.0439	1.0386	1.0340	1.0305	1.0282	1.0261	1.0226	1.0205	1.0184	1.0167	1.0153
15.00	1.1377	1.1167	1.1008	1.0893	1.0809	1.0754	1.0704	1.0667	1.0642	1.0618	1.0580	1.0557	1.0536	1.0517	1.0503
15.50	1.1770	1.1552	1.1388	1.1266	1.1178	1.1120	1.1066	1.1027	1.0999	1.0973	1.0931	1.0905	1.0883	1.0864	1.0849
16.00	1.2162	1.1937	1.1767	1.1638	1.1545	1.1484	1.1426	1.1384	1.1354	1.1325	1.1278	1.1249	1.1226	1.1206	1.1191
16.50	1.2553	1.2321	1.2144	1.2008	1.1911	1.1846	1.1784	1.1739	1.1705	1.1674	1.1622	1.1590	1.1566	1.1545	1.1529
17.00	1.2944	1.2705	1.2521	1.2378	1.2275	1.2207	1.2140	1.2092	1.2055	1.2020	1.1962	1.1928	1.1901	1.1880	1.1864
17.50	1.3334	1.3087	1.2898	1.2746	1.2638	1.2566	1.2494	1.2443	1.2402	1.2364	1.2300	1.2262	1.2234	1.2212	1.2195
18.00	1.3725	1.3470	1.3273	1.3113	1.2999	1.2923	1.2846	1.2792	1.2747	1.2705	1.2634	1.2592	1.2563	1.2540	1.2522
18.50	1.4114	1.3851	1.3648	1.3479	1.3359	1.3280	1.3197	1.3139	1.3089	1.3044	1.2966	1.2920	1.2889	1.2865	1.2847
19.00	1.4504	1.4232	1.4022	1.3844	1.3718	1.3634	1.3546	1.3484	1.3430	1.3380	1.3294	1.3245	1.3212	1.3187	1.3168
19.50	1.4893	1.4613	1.4395	1.4208	1.4075	1.3988	1.3893	1.3828	1.3769	1.3714	1.3620	1.3566	1.3531	1.3506	1.3485
20.00	1.5281	1.4993	1.4768	1.4571	1.4432	1.4340	1.4239	1.4170	1.4105	1.4046	1.3944	1.3885	1.3848	1.3822	1.3800
20.50	1.5670	1.5373	1.5140	1.4933	1.4788	1.4691	1.4584	1.4510	1.4440	1.4376	1.4265	1.4201	1.4162	1.4135	1.4112
21.00	1.6058	1.5752	1.5511	1.5294	1.5142	1.5041	1.4927	1.4849	1.4773	1.4704	1.4583	1.4515	1.4473	1.4445	1.4422
21.50	1.6446	1.6130	1.5882	1.5655	1.5495	1.5390	1.5269	1.5186	1.5104	1.5030	1.4900	1.4826	1.4782	1.4752	1.4728
22.00	1.6833	1.6509	1.6252	1.6014	1.5848	1.5738	1.5609	1.5522	1.5434	1.5355	1.5214	1.5134	1.5088	1.5057	1.5032
22.50	1.7220	1.6887	1.6622	1.6373	1.6199	1.6084	1.5948	1.5856	1.5762	1.5677	1.5525	1.5440	1.5391	1.5360	1.5333
23.00	1.7607	1.7264	1.6991	1.6732	1.6550	1.6430	1.6286	1.6189	1.6088	1.5998	1.5835	1.5744	1.5693	1.5660	1.5632
23.50	1.7994	1.7641	1.7360	1.7089	1.6900	1.6755	1.6623	1.6521	1.6413	1.6317	1.6143	1.6046	1.5992	1.5957	1.5929
24.00	1.8381	1.8018	1.7729	1.7446	1.7249	1.7118	1.6959	1.6851	1.6736	1.6634	1.6448	1.6345	1.6288	1.6253	1.6223
24.50	1.8767	1.8394	1.8097	1.7802	1.7597	1.7461	1.7294	1.7180	1.7058	1.6950	1.6752	1.6642	1.6583	1.6546	1.6515
25.00	1.9153	1.8771	1.8464	1.8158	1.7944	1.7803	1.7627	1.7508	1.7379	1.7264	1.7054	1.6937	1.6875	1.6837	1.6805
25.50	1.9539	1.9146	1.8831	1.8513	1.8291	1.8144	1.7960	1.7835	1.7698	1.7577	1.7354	1.7231	1.7165	1.7126	1.7093
26.00	1.9924	1.9522	1.9198	1.8867	1.8637	1.8484	1.8291	1.8161	1.8016	1.7888	1.7652	1.7522	1.7454	1.7413	1.7378
26.50	2.0310	1.9897	1.9564	1.9221	1.8982	1.8824	1.8622	1.8486	1.8333	1.8198	1.7949	1.7812	1.7740	1.7698	1.7662
27.00	2.0695	2.0272	1.9930	1.9574	1.9326	1.9163	1.8951	1.8810	1.8649	1.8506	1.8244	1.8100	1.8025	1.7981	1.7944
27.50	2.1080	2.0647	2.0295	1.9927	1.9670	1.9501	1.9280	1.9133	1.8963	1.8814	1.8537	1.8386	1.8308	1.8262	1.8224
28.00	2.1465	2.1021	2.0661	2.0279	2.0014	1.9838	1.9608	1.9454	1.9277	1.9119	1.8829	1.8670	1.8589	1.8542	1.8502
28.50	2.1850	2.1395	2.1025	2.0631	2.0356	2.0175	1.9935	1.9775	1.9589	1.9424	1.9119	1.8953	1.8868	1.8820	1.8779
29.00	2.2234	2.1769	2.1390	2.0983	2.0698	2.0511	2.0261	2.0094	1.9900	1.9726	1.9408	1.9234	1.9146	1.9096	1.9053
29.50	2.2619	2.2142	2.1754	2.1333	2.1040	2.0846	2.0587	2.0414	2.0210	2.0030	1.9696	1.9513	1.9422	1.9370	1.9327
30.00	2.3003	2.2516	2.2118	2.1684	2.1381	2.1180	2.0912	2.0732	2.0519	2.0331	1.9982	1.9791	1.9696	1.9643	1.9598

TABLE 34.2—EXTENDED SUKKAR-CORNELL INTEGRAL FOR BHP CALCULATION (continued)

$$\int_{0.2}^{p_{pr}} \frac{(z/p_{pr})dp_{pr}}{1+B(z/p_{pr})^2}$$

Pseudoreduced temperature for $B = 40.0$

p_{pr}	1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.8	1.9	2.0	2.2	2.4	2.6	2.8	3.0
0.20	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.50	0.0029	0.0028	0.0028	0.0027	0.0027	0.0027	0.0027	0.0027	0.0027	0.0026	0.0026	0.0026	0.0026	0.0026	0.0026
1.00	0.0150	0.0139	0.0133	0.0129	0.0127	0.0125	0.0123	0.0122	0.0122	0.0121	0.0120	0.0119	0.0119	0.0118	0.0118
1.50	0.0403	0.0341	0.0318	0.0305	0.0296	0.0290	0.0284	0.0281	0.0279	0.0276	0.0273	0.0271	0.0270	0.0268	0.0267
2.00	0.0776	0.0640	0.0582	0.0551	0.0530	0.0517	0.0505	0.0497	0.0493	0.0488	0.0482	0.0477	0.0473	0.0471	0.0468
2.50	0.1170	0.1005	0.0912	0.0858	0.0821	0.0798	0.0779	0.0765	0.0756	0.0749	0.0738	0.0730	0.0724	0.0718	0.0714
3.00	0.1565	0.1393	0.1281	0.1208	0.1156	0.1122	0.1095	0.1074	0.1061	0.1050	0.1034	0.1023	0.1013	0.1005	0.0999
3.50	0.1958	0.1787	0.1668	0.1584	0.1520	0.1477	0.1442	0.1416	0.1398	0.1383	0.1362	0.1348	0.1335	0.1324	0.1315
4.00	0.2351	0.2182	0.2062	0.1973	0.1901	0.1853	0.1812	0.1780	0.1758	0.1740	0.1714	0.1696	0.1681	0.1667	0.1656
4.50	0.2743	0.2576	0.2457	0.2367	0.2292	0.2240	0.2195	0.2159	0.2135	0.2113	0.2084	0.2063	0.2045	0.2029	0.2017
5.00	0.3133	0.2969	0.2851	0.2762	0.2686	0.2633	0.2586	0.2548	0.2521	0.2498	0.2465	0.2442	0.2422	0.2405	0.2391
5.50	0.3523	0.3360	0.3244	0.3156	0.3081	0.3028	0.2980	0.2941	0.2913	0.2889	0.2854	0.2829	0.2808	0.2790	0.2775
6.00	0.3912	0.3750	0.3634	0.3549	0.3476	0.3423	0.3376	0.3336	0.3308	0.3283	0.3247	0.3221	0.3199	0.3181	0.3166
6.50	0.4300	0.4138	0.4032	0.3939	0.3868	0.3816	0.3770	0.3731	0.3703	0.3678	0.3642	0.3616	0.3594	0.3575	0.3560
7.00	0.4687	0.4525	0.4410	0.4328	0.4258	0.4208	0.4163	0.4124	0.4097	0.4073	0.4037	0.4011	0.3989	0.3970	0.3955
7.50	0.5073	0.4910	0.4795	0.4714	0.4646	0.4597	0.4553	0.4516	0.4490	0.4466	0.4431	0.4405	0.4383	0.4365	0.4350
8.00	0.5458	0.5294	0.5179	0.5097	0.5031	0.4983	0.4941	0.4905	0.4879	0.4856	0.4819	0.4797	0.4776	0.4758	0.4743
8.50	0.5843	0.5677	0.5560	0.5479	0.5413	0.5367	0.5325	0.5290	0.5266	0.5244	0.5208	0.5187	0.5166	0.5148	0.5133
9.00	0.6227	0.6059	0.5940	0.5859	0.5793	0.5747	0.5707	0.5673	0.5650	0.5628	0.5593	0.5573	0.5553	0.5535	0.5521
9.50	0.6609	0.6439	0.6319	0.6237	0.6171	0.6125	0.6085	0.6052	0.6030	0.6009	0.5975	0.5955	0.5936	0.5918	0.5904
10.00	0.6991	0.6818	0.6696	0.6612	0.6546	0.6500	0.6461	0.6429	0.6407	0.6386	0.6353	0.6334	0.6315	0.6298	0.6284
10.50	0.7372	0.7196	0.7071	0.6987	0.6919	0.6873	0.6833	0.6802	0.6780	0.6760	0.6728	0.6710	0.6690	0.6673	0.6660
11.00	0.7753	0.7573	0.7446	0.7359	0.7290	0.7243	0.7203	0.7172	0.7150	0.7130	0.7099	0.7081	0.7062	0.7045	0.7031
11.50	0.8132	0.7949	0.7819	0.7729	0.7659	0.7611	0.7571	0.7539	0.7517	0.7498	0.7466	0.7448	0.7429	0.7412	0.7398
12.00	0.8511	0.8324	0.8190	0.8098	0.8026	0.7977	0.7936	0.7903	0.7822	0.7862	0.7830	0.7812	0.7792	0.7775	0.7762
12.50	0.8890	0.8696	0.8561	0.8466	0.8391	0.8341	0.8299	0.8265	0.8243	0.8223	0.8190	0.8171	0.8152	0.8134	0.8121
13.00	0.9268	0.9072	0.8931	0.8832	0.8755	0.8703	0.8659	0.8624	0.8602	0.8580	0.8547	0.8527	0.8507	0.8490	0.8476
13.50	0.9645	0.9445	0.9229	0.9196	0.9117	0.9063	0.9017	0.8981	0.8957	0.8935	0.8900	0.8879	0.8859	0.8841	0.8827
14.00	1.0022	0.9816	0.9667	0.9559	0.9477	0.9421	0.9373	0.9335	0.9310	0.9287	0.9250	0.9228	0.9207	0.9188	0.9174
14.50	1.0398	1.0188	1.0034	0.9921	0.9835	0.9778	0.9727	0.9588	0.9661	0.9636	0.9596	0.9572	0.9551	0.9532	0.9517
15.00	1.0774	1.0558	1.0400	1.0282	1.0193	1.0133	1.0079	1.0037	1.0009	0.9982	0.9939	0.9914	0.9891	0.9872	0.9856
15.50	1.1149	1.0928	1.0765	1.0641	1.0548	1.0486	1.0429	1.0385	1.0355	1.0326	1.0279	1.0251	1.0228	1.0208	1.0192
16.00	1.1525	1.1297	1.1129	1.1000	1.0903	1.0837	1.0777	1.0731	1.0698	1.0667	1.0616	1.0586	1.0561	1.0541	1.0525
16.50	1.1899	1.1666	1.1492	1.1357	1.1255	1.1187	1.1123	1.1075	1.1039	1.1005	1.0949	1.0917	1.0891	1.0870	1.0853
17.00	1.2274	1.2034	1.1855	1.1713	1.1607	1.1536	1.1468	1.1417	1.1378	1.1341	1.1280	1.1245	1.1218	1.1196	1.1179
17.50	1.2648	1.2402	1.2217	1.2068	1.1958	1.1884	1.1811	1.1757	1.1714	1.1675	1.1608	1.1570	1.1541	1.1519	1.1501
18.00	1.3021	1.2769	1.2579	1.2422	1.2307	1.2230	1.2152	1.2095	1.2049	1.2006	1.1934	1.1892	1.1862	1.1839	1.1820
18.50	1.3395	1.3136	1.2940	1.2776	1.2655	1.2574	1.2492	1.2432	1.2382	1.2336	1.2256	1.2211	1.2180	1.2155	1.2136
19.00	1.3768	1.3502	1.3300	1.3128	1.3002	1.2918	1.2831	1.2767	1.2713	1.2663	1.2577	1.2528	1.2494	1.2469	1.2450
19.50	1.4140	1.3868	1.3659	1.3480	1.3349	1.3261	1.3168	1.3101	1.3042	1.2988	1.2894	1.2842	1.2806	1.2780	1.2760
20.00	1.4513	1.4233	1.4019	1.3831	1.3694	1.3602	1.3504	1.3433	1.3369	1.3311	1.3210	1.3153	1.3116	1.3089	1.3068
20.50	1.4885	1.4598	1.4377	1.4181	1.4038	1.3942	1.3838	1.3763	1.3695	1.3633	1.3523	1.3462	1.3422	1.3395	1.3373
21.00	1.5257	1.4963	1.4735	1.4530	1.4381	1.4281	1.4171	1.4093	1.4019	1.3952	1.3834	1.3768	1.3727	1.3698	1.3675
21.50	1.5629	1.5327	1.5093	1.4879	1.4723	1.4620	1.4503	1.4421	1.4341	1.4270	1.4143	1.4072	1.4028	1.3999	1.3975
22.00	1.6001	1.5691	1.5450	1.5227	1.5065	1.4957	1.4834	1.4747	1.4662	1.4586	1.4449	1.4373	1.4328	1.4297	1.4272
22.50	1.6372	1.6054	1.5807	1.5574	1.5406	1.5293	1.5164	1.5072	1.4982	1.4900	1.4754	1.4673	1.4625	1.4593	1.4567
23.00	1.6743	1.6417	1.6163	1.5920	1.5746	1.5629	1.5492	1.5396	1.5300	1.5213	1.5057	1.4970	1.4920	1.4887	1.4860
23.50	1.7114	1.6780	1.6519	1.6266	1.6085	1.5963	1.5820	1.5719	1.5617	1.5525	1.5358	1.5265	1.5213	1.5178	1.5151
24.00	1.7485	1.7143	1.6874	1.6612	1.6423	1.6297	1.6146	1.6041	1.5932	1.5834	1.5657	1.5559	1.5503	1.5468	1.5439
24.50	1.7855	1.7505	1.7229	1.6947	1.6761	1.6630	1.6472	1.6362	1.6246	1.6143	1.5954	1.5850	1.5792	1.5755	1.5725
25.00	1.8226	1.7867	1.7584	1.7301	1.7098	1.6962	1.6797	1.6682	1.6559	1.6450	1.6249	1.6139	1.6078	1.6041	1.6010
25.50	1.8596	1.8229	1.7938	1.7645	1.7434	1.7293	1.7120	1.7000	1.6871	1.6755	1.6543	1.6427	1.6363	1.6324	1.6292
26.00	1.8966	1.8591	1.8292	1.7988	1.7770	1.7624	1.7443	1.7318	1.7181	1.7059	1.6836	1.6713	1.6646	1.6606	1.6572
26.50	1.9336	1.8952	1.8645	1.8331	1.8105	1.7954	1.7765	1.7634	1.7491	1.7362	1.7126	1.6997	1.6927	1.6886	1.6851
27.00	1.9705	1.9313	1.8999	1.8673	1.8439	1.8283	1.8086	1.7950	1.7799	1.7664	1.7415	1.7279	1.7207	1.7164	1.7128
27.50	2.0075	1.9674	1.9352	1.9015	1.8773	1.8612	1.8406	1.8265	1.8106	1.7965	1.7703	1.7560	1.7484	1.7440	1.7403
28.00	2.0444	2.0034	1.9704	1.9356	1.9107	1.8940	1.8726	1.8579	1.8412	1.8264	1.7989	1.7839	1.7760	1.7715	1.7676
28.50	2.0813	2.0394	2.0057	1.9697	1.9439	1.9267	1.9044	1.8892	1.8717	1.8562	1.8274	1.8116	1.8035	1.7988	1.7948
29.00	2.1182	2.0755	2.0409	2.0038	1.9771	1.9594	1.9362	1.9204	0.9021	1.8859	1.8557	1.8393	1.8308	1.8259	1.8218
29.50	2.1551	2.1114	2.0761	2.0378	2.0103	1.9920	1.9680	1.9516	1.9325	1.9155	1.8840	1.8667	1.8579	1.8529	1.8487
30.00	2.1920	2.1474	2.1112	2.0717	2.0434	2.0246	1.9996	1.9826	1.9627	1.9460	1.9120	1.8940	1.8849	1.8797	1.8754

TABLE 34.2—EXTENDED SUKKAR-CORNELL INTEGRAL FOR BHP CALCULATION (continued)

$$\int_{0.2}^{p_{pr}} \frac{(z/p_{pr})dp_{pr}}{1+B(z/p_{pr})^2}$$

Pseudoreduced temperature for $B = 45.0$

p_{pr}	1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.8	1.9	2.0	2.2	2.4	2.6	2.8	3.0
0.20	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.50	0.0026	0.0025	0.0025	0.0024	0.0024	0.0024	0.0024	0.0024	0.0024	0.0024	0.0023	0.0023	0.0023	0.0023	0.0023
1.00	0.0134	0.0124	0.0119	0.0115	0.0113	0.0111	0.0110	0.0109	0.0108	0.0108	0.0107	0.0106	0.0106	0.0105	0.0105
1.50	0.0362	0.0305	0.0284	0.0272	0.0264	0.0258	0.0254	0.0250	0.0248	0.0247	0.0244	0.0242	0.0240	0.0239	0.0238
2.00	0.0707	0.0576	0.0522	0.0494	0.0475	0.0462	0.0452	0.0445	0.0440	0.0436	0.0430	0.0426	0.0423	0.0420	0.0418
2.50	0.1076	0.0912	0.0823	0.0772	0.0738	0.0716	0.0699	0.0586	0.0678	0.0671	0.0661	0.0654	0.0648	0.0644	0.0640
3.00	0.1449	0.1273	0.1163	0.1093	0.1043	0.1012	0.0986	0.0967	0.0955	0.0944	0.0930	0.0919	0.0910	0.0903	0.0897
3.50	0.1821	0.1643	0.1523	0.1441	0.1378	0.1338	0.1304	0.1279	0.1263	0.1248	0.1229	0.1215	0.1203	0.1193	0.1185
4.00	0.2193	0.2015	0.1892	0.1803	0.1732	0.1685	0.1645	0.1614	0.1594	0.1576	0.1552	0.1534	0.1520	0.1507	0.1496
4.50	0.2565	0.2388	0.2264	0.2172	0.2096	0.2045	0.2001	0.1966	0.1942	0.1921	0.1893	0.1872	0.1855	0.1840	0.1828
5.00	0.2936	0.2760	0.2637	0.2544	0.2466	0.2412	0.2366	0.2327	0.2301	0.2278	0.2246	0.2223	0.2204	0.2187	0.2174
5.50	0.3306	0.3131	0.3009	0.2917	0.2838	0.2783	0.2735	0.2695	0.2667	0.2643	0.2608	0.2583	0.2562	0.2544	0.2530
6.00	0.3676	0.3501	0.3380	0.3289	0.3211	0.3156	0.3107	0.3066	0.3038	0.3012	0.2976	0.2949	0.2928	0.2909	0.2895
6.50	0.4045	0.3871	0.3750	0.3660	0.3583	0.3528	0.3480	0.3439	0.3410	0.3384	0.3347	0.3319	0.3297	0.3278	0.3264
7.00	0.4414	0.4239	0.4118	0.4029	0.3954	0.3900	0.3852	0.3811	0.3782	0.3757	0.3719	0.3692	0.3669	0.3650	0.3635
7.50	0.4782	0.4607	0.4486	0.4397	0.4323	0.4270	0.4223	0.4182	0.4154	0.4129	0.4092	0.4064	0.4042	0.4023	0.4008
8.00	0.5150	0.4973	0.4852	0.4763	0.4690	0.4638	0.4592	0.4552	0.4525	0.4500	0.4459	0.4436	0.4414	0.4395	0.4380
8.50	0.5517	0.5339	0.5216	0.5128	0.5055	0.5004	0.4959	0.4920	0.4893	0.4869	0.4828	0.4806	0.4785	0.4766	0.4751
9.00	0.5883	0.5704	0.5580	0.5492	0.5419	0.5368	0.5323	0.5286	0.5259	0.5235	0.5196	0.5174	0.5153	0.5135	0.5120
9.50	0.6248	0.6067	0.5942	0.5853	0.5780	0.5730	0.5686	0.5649	0.5623	0.5599	0.5561	0.5540	0.5519	0.5501	0.5486
10.00	0.6613	0.6430	0.6304	0.6214	0.6140	0.6090	0.6046	0.6009	0.5984	0.5961	0.5923	0.5903	0.5882	0.5864	0.5650
10.50	0.6978	0.6792	0.6664	0.6573	0.6498	0.6447	0.6404	0.6367	0.6342	0.6320	0.6283	0.6262	0.6242	0.6224	0.6210
11.00	0.7342	0.7153	0.7023	0.6930	0.6854	0.6803	0.6759	0.6723	0.6698	0.6676	0.6639	0.6619	0.6598	0.6580	0.6566
11.50	0.7705	0.7514	0.7381	0.7286	0.7209	0.7157	0.7113	0.7076	0.7051	0.7029	0.6993	0.6972	0.6952	0.6934	0.6920
12.00	0.8068	0.7874	0.7738	0.7641	0.7562	0.7509	0.7464	0.7427	0.7402	0.7380	0.7343	0.7323	0.7302	0.7284	0.7270
12.50	0.8430	0.8233	0.8094	0.7994	0.7914	0.7860	0.7814	0.7776	0.7751	0.7728	0.7690	0.7670	0.7649	0.7680	0.7616
13.00	0.8792	0.8591	0.8449	0.8347	0.8264	0.8209	0.8161	0.8122	0.8097	0.8073	0.8035	0.8013	0.7992	0.7974	0.7959
13.50	0.9153	0.8949	0.8804	0.8698	0.8613	0.8556	0.8507	0.8467	0.8440	0.8416	0.8376	0.8354	0.8332	0.8313	0.8299
14.00	0.9514	0.9306	0.9157	0.9048	0.8961	0.8902	0.8851	0.8809	0.8782	0.8756	0.8715	0.8691	0.8669	0.8650	0.8635
14.50	0.9875	0.9663	0.9510	0.9396	0.9307	0.9246	0.9193	0.9150	0.9121	0.9094	0.9050	0.9025	0.9002	0.8983	0.8968
15.00	1.0235	1.0019	0.9863	0.9744	0.9652	0.9589	0.9533	0.9489	0.9458	0.9429	0.9382	0.9356	0.9332	0.9312	0.9297
15.50	1.0595	1.0374	1.0214	1.0091	0.9995	0.9931	0.9872	0.9825	0.9793	0.9762	0.9712	0.9684	0.9660	0.9639	0.9623
16.00	1.0955	1.0729	1.0565	1.0437	1.0338	1.0271	1.0209	1.0160	1.0125	1.0093	1.0039	1.0009	0.9984	0.9963	0.9946
16.50	1.1315	1.1084	1.0915	1.0782	1.0679	1.0609	1.0544	1.0494	1.0456	1.0422	1.0364	1.0331	1.0305	1.0283	1.0266
17.00	1.1674	1.1438	1.1265	1.1126	1.1019	1.0947	1.0878	1.0825	1.0785	1.0748	1.0685	1.0650	1.0623	1.0600	1.0583
17.50	1.2032	1.1791	1.1614	1.1469	1.1358	1.1283	1.1211	1.1155	1.1112	1.1072	1.1005	1.0967	1.0938	1.0915	1.0897
18.00	1.2391	1.2145	1.1962	1.1811	1.1696	1.1619	1.1542	1.1484	1.1437	1.1394	1.1321	1.1281	1.1250	1.1227	1.1208
18.50	1.2749	1.2497	1.2310	1.2153	1.2033	1.1953	1.1872	1.1811	1.1761	1.1715	1.1636	1.1592	1.1560	1.1536	1.1517
19.00	1.3107	1.2850	1.2658	1.2494	1.2370	1.2286	1.2200	1.2136	1.2082	1.2033	1.1948	1.1901	1.1867	1.1842	1.1823
19.50	1.3465	1.3202	1.3005	1.2834	1.2705	1.2618	1.2528	1.2460	1.2403	1.2350	1.2258	1.2207	1.2172	1.2146	1.2126
20.00	1.3823	1.3554	1.3351	1.3173	1.3039	1.2949	1.2854	1.2783	1.2721	1.2665	1.2566	1.2511	1.2474	1.2447	1.2426
20.50	1.4180	1.3905	1.3697	1.3512	1.3373	1.3279	1.3179	1.3105	1.3038	1.2978	1.2871	1.2812	1.2774	1.2746	1.2724
21.00	1.4538	1.4256	1.4043	1.3850	1.3706	1.3608	1.3503	1.3425	1.3354	1.3290	1.3175	1.3112	1.3071	1.3043	1.3020
21.50	1.4895	1.4607	1.4388	1.4187	1.4038	1.3937	1.3825	1.3744	1.3668	1.3599	1.3477	1.3409	1.3367	1.3337	1.3314
22.00	1.5251	1.4958	1.4733	1.4524	1.4369	1.4264	1.4147	1.4062	1.3981	1.3908	1.3776	1.3704	1.3660	1.3629	1.3605
22.50	1.5608	1.5308	1.5077	1.4860	1.4699	1.4591	1.4468	1.4379	1.4292	1.4215	1.4074	1.3997	1.3951	1.3919	1.3894
23.00	1.5965	1.5658	1.5421	1.5196	1.5029	1.4916	1.4788	1.4694	1.4603	1.4520	1.4371	1.4288	1.4239	1.4207	1.4181
23.50	1.6321	1.6008	1.5765	1.5531	1.5358	1.5242	1.5106	1.5009	1.4912	1.4824	1.4665	1.4577	1.4526	1.4493	1.4466
24.00	1.6677	1.6357	1.6108	1.5866	1.5687	1.5566	1.5424	1.5323	1.5219	1.5127	1.4958	1.4865	1.4811	1.4776	1.4748
24.50	1.7033	1.6706	1.6451	1.6200	1.6015	1.5890	1.5741	1.5635	1.5526	1.5428	1.5249	1.5150	1.5094	1.5058	1.5029
25.00	1.7389	1.7055	1.6794	1.6534	1.6342	1.6212	1.6057	1.5947	1.5831	1.5728	1.5538	1.5434	1.5375	1.5338	1.5308
25.50	1.7745	1.7404	1.7136	1.6867	1.6668	1.6535	1.6373	1.6247	1.6136	1.6027	1.5826	1.5716	1.5655	1.5617	1.5585
26.00	1.8100	1.7752	1.7478	1.7200	1.6995	1.6856	1.6687	1.6567	1.6439	1.6324	1.6112	1.5996	1.5933	1.5893	1.5861
26.50	1.8456	1.8101	1.7820	1.7532	1.7320	1.7177	1.7001	1.6876	1.6741	1.6621	1.6397	1.6275	1.6209	1.6168	1.6134
27.00	1.8811	1.8449	1.8162	1.7864	1.7645	1.7498	1.7314	1.7184	1.7042	1.6916	1.6681	1.6552	1.6483	1.6441	1.6406
27.50	1.9166	1.8797	1.8503	1.8195	1.7969	1.7817	1.7626	1.7491	1.7343	1.7210	1.6963	1.6828	1.6756	1.6712	1.6677
28.00	1.9521	1.9144	1.8844	1.8526	1.8293	1.8136	1.7937	1.7798	1.7642	1.7503	1.7244	1.7102	1.7027	1.6982	1.6945
28.50	1.9876	1.9492	1.9184	1.8857	1.8617	1.8455	1.8248	1.8103	1.7940	1.7795	1.7523	1.7375	1.7297	1.7251	1.7212
29.00	2.0231	1.9839	1.9525	1.9187	1.8940	1.8773	1.8558	1.8408	1.8238	1.8086	1.7801	1.7646	1.7565	1.7518	1.7478
29.50	2.0586	2.0186	1.9865	1.9517	1.9262	1.9091	1.8868	1.8712	1.8534	1.8376	1.8078	1.7916	1.7832	1.7783	1.7742
30.00	2.0941	2.0533	2.0205	1.9847	1.9584	1.9408	1.9176	1.9016	1.8830	1.8664	1.8354	1.8184	1.8097	1.8047	1.8005

TABLE 34.2—EXTENDED SUKKAR-CORNELL INTEGRAL FOR BHP CALCULATION (continued)

$$\int_{0.2}^{p_{pr}} \frac{(z/p_{pr})dp_{pr}}{1+B(z/p_{pr})^2}$$

Pseudoreduced temperature for $B = 50.0$

p_{pr}	1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.8	1.9	2.0	2.2	2.4	2.6	2.8	3.0
0.20	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.50	0.0023	0.0023	0.0022	0.0022	0.0022	0.0022	0.0021	0.0021	0.0021	0.0021	0.0021	0.0021	0.0021	0.0021	0.0021
1.00	0.0121	0.0111	0.0107	0.0104	0.0102	0.0100	0.0099	0.0098	0.0098	0.0097	0.0096	0.0096	0.0095	0.0095	0.0095
1.50	0.0328	0.0276	0.0257	0.0246	0.0238	0.0233	0.0229	0.0226	0.0224	0.0222	0.0220	0.0218	0.0217	0.0216	0.0215
2.00	0.0649	0.0524	0.0474	0.0447	0.0430	0.0418	0.0409	0.0402	0.0398	0.0395	0.0389	0.0385	0.0382	0.0380	0.0378
2.50	0.0997	0.0835	0.0750	0.0702	0.0670	0.0650	0.0634	0.0622	0.0615	0.0608	0.0599	0.0593	0.0587	0.0583	0.0579
3.00	0.1350	0.1173	0.1066	0.0998	0.0951	0.0921	0.0897	0.0879	0.0868	0.0858	0.0844	0.0835	0.0827	0.0820	0.0814
3.50	0.1703	0.1521	0.1402	0.1322	0.1261	0.1222	0.1191	0.1167	0.1151	0.1138	0.1119	0.1106	0.1095	0.1085	0.1078
4.00	0.2057	0.1873	0.1749	0.1660	0.1591	0.1545	0.1507	0.1477	0.1457	0.1440	0.1417	0.1401	0.1387	0.1375	0.1365
4.50	0.2410	0.2226	0.2101	0.2008	0.1933	0.1882	0.1839	0.1804	0.1781	0.1761	0.1734	0.1714	0.1697	0.1633	0.1671
5.00	0.2763	0.2579	0.2454	0.2359	0.2281	0.2227	0.2181	0.2143	0.2117	0.2094	0.2063	0.2040	0.2022	0.2006	0.1993
5.50	0.3116	0.2933	0.2807	0.2712	0.2632	0.2577	0.2529	0.2488	0.2461	0.2436	0.2402	0.2377	0.2357	0.2339	0.2326
6.00	0.3469	0.3285	0.3161	0.3066	0.2985	0.2929	0.2880	0.2838	0.2809	0.2784	0.2747	0.2721	0.2700	0.2681	0.2667
6.50	0.3821	0.3638	0.3513	0.3419	0.3339	0.3282	0.3233	0.3190	0.3161	0.3135	0.3097	0.3069	0.3048	0.3029	0.3014
7.00	0.4173	0.3990	0.3865	0.3772	0.3692	0.3636	0.3587	0.3544	0.3514	0.3488	0.3450	0.3421	0.3399	0.3380	0.3365
7.50	0.4525	0.4341	0.4216	0.4123	0.4044	0.3989	0.3940	0.3897	0.3868	0.3841	0.3803	0.3774	0.3752	0.3733	0.3718
8.00	0.4876	0.4692	0.4567	0.4474	0.4395	0.4340	0.4292	0.4250	0.4221	0.4194	0.4151	0.4128	0.4105	0.4086	0.4071
8.50	0.5227	0.5042	0.4916	0.4823	0.4745	0.4690	0.4643	0.4601	0.4573	0.4547	0.4504	0.4481	0.4458	0.4439	0.4424
9.00	0.5577	0.5391	0.5264	0.5171	0.5093	0.5039	0.4992	0.4951	0.4923	0.4897	0.4855	0.4832	0.4810	0.4791	0.4777
9.50	0.5927	0.5739	0.5612	0.5518	0.5440	0.5386	0.5340	0.5299	0.5271	0.5246	0.5204	0.5182	0.5160	0.5142	0.5127
10.00	0.6277	0.6087	0.5959	0.5864	0.5786	0.5732	0.5685	0.5645	0.5618	0.5593	0.5552	0.5530	0.5508	0.5490	0.5475
10.50	0.6626	0.6435	0.6304	0.6209	0.6130	0.6076	0.6029	0.5990	0.5962	0.5938	0.5897	0.5875	0.5854	0.5835	0.5821
11.00	0.6974	0.6781	0.6649	0.6553	0.6473	0.6418	0.6372	0.6332	0.6305	0.6280	0.6240	0.6219	0.6197	0.6179	0.6164
11.50	0.7323	0.7127	0.6994	0.6896	0.6815	0.6759	0.6712	0.6672	0.6645	0.6621	0.6581	0.6559	0.6537	0.6519	0.6505
12.00	0.7670	0.7473	0.7337	0.7237	0.7155	0.7099	0.7051	0.7011	0.6984	0.6959	0.6919	0.6897	0.6875	0.6857	0.6842
12.50	0.8018	0.7818	0.7680	0.7578	0.7494	0.7437	0.7388	0.7347	0.7320	0.7295	0.7254	0.7232	0.7210	0.7192	0.7177
13.00	0.8365	0.8163	0.8022	0.7917	0.7832	0.7774	0.7724	0.7682	0.7654	0.7629	0.7587	0.7565	0.7542	0.7523	0.7509
13.50	0.8712	0.8507	0.8363	0.8256	0.8169	0.8109	0.8058	0.8015	0.7987	0.7960	0.7917	0.7894	0.7872	0.7852	0.7838
14.00	0.9059	0.8850	0.8704	0.8594	0.8504	0.8443	0.8391	0.8347	0.8317	0.8290	0.8245	0.8221	0.8198	0.8178	0.8163
14.50	0.9405	0.9193	0.9044	0.8930	0.8839	0.8776	0.8722	0.8576	0.8645	0.8617	0.8570	0.8545	0.8521	0.8502	0.8486
15.00	0.9751	0.9536	0.9384	0.9266	0.9172	0.9108	0.9051	0.9004	0.8972	0.8942	0.8893	0.8866	0.8842	0.8822	0.8806
15.50	1.0097	0.9878	0.9722	0.9601	0.9504	0.9438	0.9379	0.9331	0.9297	0.9265	0.9213	0.9185	0.9160	0.9139	0.9123
16.00	1.0442	1.0220	1.0061	0.9935	0.9836	0.9768	0.9706	0.9656	0.9620	0.9586	0.9531	0.9501	0.9475	0.9454	0.9438
16.50	1.0788	1.0561	1.0399	1.0269	1.0166	1.0096	1.0031	0.9979	0.9941	0.9906	0.9847	0.9814	0.9788	0.9766	0.9749
17.00	1.1133	1.0902	1.0736	1.0601	1.0495	1.0423	1.0355	1.0301	1.0260	1.0223	1.0160	1.0125	1.0097	1.0075	1.0058
17.50	1.1477	1.1243	1.1073	1.0933	1.0824	1.0749	1.0678	1.0621	1.0578	1.0538	1.0471	1.0434	1.0405	1.0382	1.0364
18.00	1.1822	1.1583	1.1409	1.1264	1.1151	1.1074	1.0999	1.0940	1.0894	1.0852	1.0779	1.0740	1.0709	1.0686	1.0668
18.50	1.2167	1.1923	1.1745	1.1595	1.1478	1.1398	1.1320	1.1258	1.1209	1.1164	1.1086	1.1043	1.1012	1.0988	1.0969
19.00	1.2511	1.2263	1.2081	1.1925	1.1804	1.1721	1.1639	1.1575	1.1522	1.1474	1.1390	1.1345	1.1312	1.1287	1.1268
19.50	1.2855	1.2602	1.2416	1.2254	1.2129	1.2044	1.1957	1.1890	1.1834	1.1783	1.1693	1.1644	1.1609	1.1584	1.1564
20.00	1.3199	1.2942	1.2751	1.2583	1.2453	1.2365	1.2274	1.2204	1.2144	1.2090	1.1993	1.1941	1.1905	1.1878	1.1858
20.50	1.3542	1.3280	1.3085	1.2911	1.2777	1.2686	1.2590	1.2517	1.2453	1.2395	1.2292	1.2236	1.2198	1.2171	1.2149
21.00	1.3886	1.3619	1.3419	1.3238	1.3100	1.3005	1.2905	1.2829	1.2761	1.2699	1.2589	1.2528	1.2489	1.2461	1.2439
21.50	1.4229	1.3957	1.3753	1.3565	1.3422	1.3324	1.3219	1.3140	1.3067	1.3001	1.2884	1.2810	1.2778	1.2749	1.2726
22.00	1.4573	1.4295	1.4086	1.3892	1.3743	1.3643	1.3532	1.3449	1.3372	1.3302	1.3177	1.3108	1.3065	1.3035	1.3011
22.50	1.4916	1.4633	1.4419	1.4218	1.4064	1.3960	1.3844	1.3758	1.3676	1.3602	1.3468	1.3395	1.3350	1.3319	1.3295
23.00	1.5259	1.4971	1.4752	1.4543	1.4385	1.4277	1.4155	1.4066	1.3979	1.3900	1.3758	1.3680	1.3633	1.3601	1.3576
23.50	1.5602	1.5308	1.5084	1.4868	1.4704	1.4593	1.4466	1.4372	1.4280	1.4197	1.4046	1.3964	1.3914	1.3881	1.3855
24.00	1.5944	1.5646	1.5416	1.5193	1.5024	1.4908	1.4775	1.4678	1.4581	1.4493	1.4333	1.4245	1.4193	1.4160	1.4133
24.50	1.6287	1.5983	1.5748	1.5517	1.5342	1.5223	1.5084	1.4983	1.4880	1.4788	1.4618	1.4525	1.4471	1.4436	1.4408
25.00	1.6629	1.6319	1.6079	1.5841	1.5660	1.5537	1.5392	1.5287	1.5178	1.5081	1.4902	1.4803	1.4747	1.4711	1.4682
25.50	1.6972	1.6656	1.6410	1.6164	1.5978	1.5851	1.5700	1.5590	1.5476	1.5373	1.5184	1.5080	1.5021	1.4984	1.4954
26.00	1.7314	1.6992	1.6741	1.6487	1.6295	1.6164	1.6006	1.5892	1.5772	1.5664	1.5465	1.5355	1.5294	1.5256	1.5225
26.50	1.7656	1.7329	1.7072	1.6809	1.6611	1.6476	1.6312	1.6194	1.6068	1.5954	1.5744	1.5629	1.5565	1.5526	1.5494
27.00	1.7998	1.7665	1.7403	1.7131	1.6927	1.6788	1.6617	1.6494	1.6362	1.6243	1.6022	1.5901	1.5835	1.5794	1.5761
27.50	1.8340	1.8001	1.7733	1.7453	1.7243	1.7100	1.6922	1.6794	1.6656	1.6531	1.6299	1.6172	1.6103	1.6061	1.6027
28.00	1.8682	1.8337	1.8063	1.7775	1.7558	1.7410	1.7226	1.7094	1.6948	1.6818	1.6574	1.6441	1.6369	1.6326	1.6291
28.50	1.9024	1.8672	1.8393	1.8096	1.7872	1.7721	1.7529	1.7392	1.7240	1.7104	1.6849	1.6709	1.6634	1.6590	1.6553
29.00	1.9366	1.9008	1.8722	1.8416	1.8187	1.8030	1.7831	1.7690	1.7531	1.7309	1.7122	1.6976	1.6898	1.6853	1.6815
29.50	1.9707	1.9341	1.9052	1.8737	1.8500	1.8340	1.8133	1.7987	1.7821	1.7673	1.7394	1.7241	1.7160	1.7114	1.7076
30.00	2.0049	1.9678	1.9381	1.9057	1.8814	1.8649	1.8435	1.8284	1.8111	1.7956	1.7664	1.7505	1.7421	1.7373	1.7333

TABLE 34.2—EXTENDED SUKKAR-CORNELL INTEGRAL FOR BHP CALCULATION (continued)

$$\int_{0.2}^{p_{pr}} \frac{(z/p_{pr})dp_{pr}}{1+B(z/p_{pr})^2}$$

Pseudoreduced temperature for $B = 60.0$

p_{pr}	1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.8	1.9	2.0	2.2	2.4	2.6	2.8	3.0
0.20	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.50	0.0019	0.0019	0.0019	0.0018	0.0018	0.0018	0.0018	0.0018	0.0018	0.0018	0.0018	0.0018	0.0017	0.0017	0.0017
1.00	0.0101	0.0093	0.0089	0.0087	0.0085	0.0084	0.0083	0.0082	0.0081	0.0081	0.0080	0.0080	0.0080	0.0079	0.0079
1.50	0.0277	0.0232	0.0215	0.0206	0.0200	0.0195	0.0192	0.0189	0.0188	0.0186	0.0184	0.0183	0.0181	0.0181	0.0180
2.00	0.0559	0.0443	0.0399	0.0376	0.0361	0.0351	0.0343	0.0338	0.0334	0.0331	0.0326	0.0323	0.0321	0.0319	0.0317
2.50	0.0870	0.0715	0.0637	0.0594	0.0566	0.0549	0.0535	0.0524	0.0518	0.0512	0.0504	0.0499	0.0494	0.0490	0.0487
3.00	0.1189	0.1014	0.0913	0.0851	0.0808	0.0781	0.0760	0.0745	0.0734	0.0726	0.0714	0.0705	0.0698	0.0692	0.0687
3.50	0.1509	0.1325	0.1211	0.1135	0.1079	0.1043	0.1014	0.0993	0.0979	0.0966	0.0950	0.0939	0.0928	0.0920	0.0913
4.00	0.1831	0.1642	0.1521	0.1435	0.1369	0.1326	0.1291	0.1263	0.1245	0.1229	0.1209	0.1194	0.1181	0.1170	0.1161
4.50	0.2153	0.1962	0.1837	0.1745	0.1672	0.1624	0.1583	0.1551	0.1529	0.1510	0.1485	0.1466	0.1451	0.1438	0.1428
5.00	0.2475	0.2283	0.2157	0.2062	0.1984	0.1931	0.1887	0.1850	0.1826	0.1804	0.1775	0.1753	0.1736	0.1721	0.1709
5.50	0.2798	0.2606	0.2479	0.2382	0.2301	0.2245	0.2198	0.2158	0.2132	0.2108	0.2075	0.2051	0.2032	0.2016	0.2003
6.00	0.3120	0.2928	0.2801	0.2703	0.2620	0.2563	0.2515	0.2472	0.2444	0.2419	0.2383	0.2357	0.2337	0.2320	0.2306
6.50	0.3443	0.3251	0.3124	0.3026	0.2942	0.2884	0.2834	0.2791	0.2761	0.2735	0.2697	0.2670	0.2648	0.2630	0.2616
7.00	0.3766	0.3574	0.3446	0.3348	0.3264	0.3206	0.3156	0.3111	0.3081	0.3054	0.3015	0.2986	0.2964	0.2946	0.2932
7.50	0.4088	0.3896	0.3769	0.3671	0.3587	0.3529	0.3478	0.3433	0.3403	0.3375	0.3336	0.3306	0.3284	0.3265	0.3251
8.00	0.4411	0.4219	0.4091	0.3994	0.3910	0.3851	0.3801	0.3756	0.3725	0.3697	0.3651	0.3628	0.3605	0.3586	0.3572
8.50	0.4734	0.4541	0.4413	0.4316	0.4232	0.4174	0.4123	0.4079	0.4048	0.4020	0.3974	0.3951	0.3928	0.3909	0.3894
9.00	0.5056	0.4863	0.4735	0.4637	0.4554	0.4496	0.4445	0.4401	0.4370	0.4343	0.4297	0.4273	0.4251	0.4231	0.4217
9.50	0.5378	0.5185	0.5056	0.4958	0.4875	0.4817	0.4767	0.4722	0.4692	0.4665	0.4619	0.4596	0.4573	0.4554	0.4539
10.00	0.5701	0.5507	0.5377	0.5279	0.5195	0.5137	0.5087	0.5043	0.5013	0.4985	0.4940	0.4917	0.4894	0.4875	0.4861
10.50	0.6023	0.5828	0.5698	0.5599	0.5515	0.5457	0.5407	0.5363	0.5333	0.5305	0.5260	0.5237	0.5215	0.5196	0.5181
11.00	0.6344	0.6149	0.6018	0.5918	0.5833	0.5775	0.5725	0.5681	0.5651	0.5624	0.5579	0.5556	0.5534	0.5515	0.5500
11.50	0.6666	0.6469	0.6337	0.6237	0.6151	0.6093	0.6042	0.5998	0.5968	0.5941	0.5896	0.5873	0.5851	0.5832	0.5818
12.00	0.6987	0.6790	0.6656	0.6555	0.6469	0.6409	0.6359	0.6314	0.6284	0.6257	0.6212	0.6189	0.6166	0.6148	0.6133
12.50	0.7309	0.7110	0.6975	0.6872	0.6785	0.6725	0.6674	0.6629	0.6599	0.6571	0.6526	0.6503	0.6480	0.6461	0.6446
13.00	0.7630	0.7429	0.7293	0.7189	0.7101	0.7040	0.6986	0.6943	0.6912	0.6884	0.6838	0.6815	0.6792	0.6773	0.6758
13.50	0.7951	0.7749	0.7611	0.7505	0.7415	0.7354	0.7301	0.7255	0.7224	0.7196	0.7149	0.7125	0.7101	0.7032	0.7067
14.00	0.8272	0.8068	0.7929	0.7820	0.7730	0.7667	0.7613	0.7566	0.7534	0.7505	0.7457	0.7432	0.7409	0.7389	0.7374
14.50	0.8592	0.8387	0.8246	0.8135	0.8043	0.7979	0.7924	0.7876	0.7843	0.7813	0.7764	0.7738	0.7714	0.7694	0.7679
15.00	0.8913	0.8705	0.8562	0.8449	0.8355	0.8291	0.8233	0.8184	0.8151	0.8120	0.8069	0.8042	0.8017	0.7997	0.7982
15.50	0.9233	0.9024	0.8879	0.8763	0.8667	0.8601	0.8542	0.8492	0.8457	0.8425	0.8371	0.8343	0.8318	0.8298	0.8282
16.00	0.9554	0.9342	0.9195	0.9076	0.8978	0.8911	0.8850	0.8798	0.8762	0.8728	0.8672	0.8643	0.8617	0.8596	0.8580
16.50	0.9874	0.9660	0.9510	0.9389	0.9288	0.9219	0.9156	0.9103	0.9065	0.9030	0.8971	0.8940	0.8914	0.8892	0.8876
17.00	1.0194	0.9977	0.9826	0.9701	0.9598	0.9527	0.9462	0.9408	0.9368	0.9331	0.9269	0.9236	0.9208	0.9186	0.9170
17.50	1.0514	1.0295	1.0141	1.0012	0.9907	0.9835	0.9767	0.9711	0.9668	0.9630	0.9564	0.9529	0.9501	0.9478	0.9461
18.00	1.0834	1.0612	1.0455	1.0323	1.0215	1.0141	1.0070	1.0013	0.9968	0.9928	0.9858	0.9820	0.9791	0.9768	0.9751
18.50	1.1153	1.0929	1.0769	1.0634	1.0523	1.0447	1.0373	1.0313	1.0267	1.0224	1.0150	1.0110	1.0080	1.0056	1.0038
19.00	1.1473	1.1246	1.1083	1.0944	1.0830	1.0752	1.0675	1.0613	1.0564	1.0519	1.0440	1.0398	1.0366	1.0342	1.0324
19.50	1.1792	1.1562	1.1397	1.1253	1.1137	1.1056	1.0976	1.0912	1.0860	1.0812	1.0728	1.0683	1.0651	1.0626	1.0607
20.00	1.2112	1.1879	1.1711	1.1562	1.1443	1.1360	1.1277	1.1210	1.1155	1.1104	1.1015	1.0967	1.0933	1.0908	1.0889
20.50	1.2431	1.2195	1.2024	1.1871	1.1748	1.1663	1.1576	1.1507	1.1449	1.1395	1.1301	1.1250	1.1214	1.1188	1.1168
21.00	1.2750	1.2511	1.2337	1.2179	1.2053	1.1965	1.1875	1.1803	1.1741	1.1685	1.1584	1.1530	1.1493	1.1466	1.1446
21.50	1.3069	1.2827	1.2650	1.2487	1.2357	1.2267	1.2173	1.2099	1.2033	1.1974	1.1867	1.1809	1.1770	1.1743	1.1721
22.00	1.3388	1.3143	1.2962	1.2795	1.2661	1.2568	1.2470	1.2393	1.2324	1.2261	1.2147	1.2086	1.2046	1.2018	1.1995
22.50	1.3707	1.3458	1.3274	1.3102	1.2964	1.2869	1.2766	1.2687	1.2614	1.2547	1.2427	1.2361	1.2319	1.2291	1.2268
23.00	1.4026	1.3774	1.3586	1.3409	1.3267	1.3169	1.3062	1.2979	1.2902	1.2832	1.2705	1.2635	1.2592	1.2562	1.2538
23.50	1.4344	1.4089	1.3898	1.3715	1.3569	1.3469	1.3357	1.3271	1.3190	1.3116	1.2981	1.2908	1.2862	1.2832	1.2807
24.00	1.4663	1.4404	1.4210	1.4021	1.3871	1.3768	1.3652	1.3563	1.3477	1.3399	1.3256	1.3179	1.3131	1.3100	1.3074
24.50	1.4982	1.4719	1.4521	1.4327	1.4173	1.4066	1.3945	1.3853	1.3763	1.3681	1.3530	1.3448	1.3399	1.3366	1.3340
25.00	1.5300	1.5034	1.4832	1.4632	1.4474	1.4364	1.4238	1.4143	1.4048	1.3962	1.3803	1.3716	1.3664	1.3631	1.3604
25.50	1.5619	1.5349	1.5143	1.4937	1.4774	1.4662	1.4531	1.4432	1.4332	1.4242	1.4074	1.3983	1.3929	1.3895	1.3867
26.00	1.5937	1.5664	1.5454	1.5242	1.5075	1.4959	1.4823	1.4721	1.4616	1.4521	1.4344	1.4248	1.4192	1.4157	1.4128
26.50	1.6255	1.5978	1.5765	1.5547	1.5374	1.5255	1.5114	1.5008	1.4898	1.4799	1.4613	1.4512	1.4454	1.4417	1.4388
27.00	1.6574	1.6292	1.6075	1.5851	1.5674	1.5552	1.5405	1.5295	1.5180	1.5076	1.4881	1.4775	1.4714	1.4677	1.4646
27.50	1.6892	1.6607	1.6385	1.6155	1.5973	1.5847	1.5695	1.5582	1.5461	1.5353	1.5148	1.5036	1.4973	1.4935	1.4903
28.00	1.7210	1.6921	1.6695	1.6459	1.6272	1.6143	1.5985	1.5868	1.5742	1.5628	1.5413	1.5296	1.5231	1.5191	1.5159
28.50	1.7528	1.7235	1.7005	1.6762	1.6570	1.6438	1.6274	1.6153	1.6021	1.5903	1.5678	1.5555	1.5487	1.5447	1.5413
29.00	1.7846	1.7549	1.7315	1.7065	1.6868	1.6732	1.6563	1.6436	1.6300	1.6176	1.5941	1.5813	1.5742	1.5701	1.5666
29.50	1.8164	1.7863	1.7625	1.7368	1.7166	1.7076	1.6851	1.6722	1.6579	1.6449	1.6204	1.6070	1.5997	1.5954	1.5918
30.00	1.8482	1.8177	1.7934	1.7671	1.7463	1.7320	1.7139	1.7005	1.6856	1.6722	1.6465	1.6325	1.6249	1.6205	1.6168

TABLE 34.2—EXTENDED SUKKAR-CORNELL INTEGRAL FOR BHP CALCULATION (continued)

$$\int_{0.2}^{p_{pr}} \frac{(z/p_{pr})dp_{pr}}{1+B(z/p_{pr})^2}$$

Pseudoreduced temperature for $B = 70.0$

p_{pr}	1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.8	1.9	2.0	2.2	2.4	2.6	2.8	3.0
0.20	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.50	0.0017	0.0016	0.0016	0.0016	0.0016	0.0015	0.0015	0.0015	0.0015	0.0015	0.0015	0.0015	0.0015	0.0015	0.0015
1.00	0.0087	0.0080	0.0077	0.0074	0.0073	0.0072	0.0071	0.0070	0.0070	0.0070	0.0069	0.0069	0.0068	0.0068	0.0068
1.50	0.0240	0.0199	0.0185	0.0177	0.0172	0.0168	0.0165	0.0163	0.0161	0.0160	0.0158	0.0157	0.0156	0.0155	0.0154
2.00	0.0491	0.0385	0.0345	0.0325	0.0312	0.0303	0.0296	0.0291	0.0288	0.0285	0.0281	0.0278	0.0276	0.0274	0.0273
2.50	0.0772	0.0625	0.0554	0.0515	0.0490	0.0475	0.0462	0.0453	0.0448	0.0443	0.0435	0.0431	0.0426	0.0423	0.0420
3.00	0.1063	0.0894	0.0799	0.0742	0.0703	0.0679	0.0660	0.0646	0.0637	0.0629	0.0618	0.0611	0.0604	0.0599	0.0595
3.50	0.1356	0.1175	0.1066	0.0994	0.0943	0.0910	0.0884	0.0864	0.0851	0.0840	0.0825	0.0815	0.0806	0.0798	0.0792
4.00	0.1651	0.1464	0.1346	0.1264	0.1202	0.1162	0.1129	0.1104	0.1087	0.1073	0.1054	0.1040	0.1029	0.1018	0.1010
4.50	0.1947	1.1756	0.1634	0.1545	0.1475	0.1429	0.1391	0.1360	0.1340	0.1322	0.1299	0.1282	0.1268	0.1256	0.1246
5.00	0.2243	1.2050	0.1926	0.1833	0.1756	0.1706	0.1664	0.1629	0.1606	0.1585	0.1558	0.1538	0.1522	0.1508	0.1497
5.50	0.2540	0.2347	0.2221	0.2125	0.2045	0.1991	0.1946	0.1907	0.1881	0.1859	0.1827	0.1805	0.1787	0.1772	0.1760
6.00	0.2838	0.2644	0.2517	0.2420	0.2337	0.2281	0.2233	0.2192	0.2164	0.2140	0.2106	0.2081	0.2061	0.2045	0.2032
6.50	0.3135	0.2941	0.2815	0.2716	0.2632	0.2574	0.2525	0.2482	0.2453	0.2427	0.2390	0.2363	0.2343	0.2326	0.2313
7.00	0.3433	0.3239	0.3113	0.3014	0.2929	0.2870	0.2820	0.2775	0.2745	0.2718	0.2680	0.2652	0.2630	0.2613	0.2599
7.50	0.3732	0.3538	0.3411	0.3312	0.3226	0.3167	0.3116	0.3071	0.3040	0.3013	0.2973	0.2944	0.2922	0.2904	0.2890
8.00	0.4030	0.3836	0.3710	0.3611	0.3525	0.3465	0.3414	0.3368	0.3337	0.3309	0.3262	0.3239	0.3217	0.3198	0.3184
8.50	0.4328	0.4135	0.4009	0.3909	0.3824	0.3764	0.3713	0.3667	0.3635	0.3607	0.3560	0.3536	0.3514	0.3495	0.3481
9.00	0.4627	0.4434	0.4307	0.4208	0.4122	0.4063	0.4011	0.3965	0.3934	0.3905	0.3858	0.3834	0.3812	0.3793	0.3779
9.50	0.4926	0.4733	0.4606	0.4507	0.4421	0.4362	0.4310	0.4264	0.4233	0.4204	0.4157	0.4133	0.4110	0.4092	0.4077
10.00	0.5225	0.5031	0.4905	0.4805	0.4720	0.4660	0.4609	0.4563	0.4531	0.4503	0.4456	0.4432	0.4409	0.4390	0.4376
10.50	0.5523	0.5330	0.5203	0.5104	0.5018	0.4958	0.4907	0.4861	0.4830	0.4801	0.4754	0.4730	0.4708	0.4689	0.4675
11.00	0.5822	0.5629	0.5502	0.5402	0.5316	0.5256	0.5204	0.5159	0.5127	0.5099	0.5052	0.5028	0.5005	0.4987	0.4972
11.50	0.6121	0.5927	0.5800	0.5700	0.5613	0.5553	0.5502	0.5456	0.5424	0.5396	0.5349	0.5325	0.5303	0.5284	0.5270
12.00	0.6420	0.6226	0.6098	0.5997	0.5910	0.5850	0.5798	0.5752	0.5721	0.5692	0.5645	0.5621	0.5599	0.5580	0.5566
12.50	0.6718	0.6524	0.6396	0.6294	0.6207	0.6146	0.6094	0.6047	0.6016	0.5987	0.5940	0.5916	0.5893	0.5875	0.5860
13.00	0.7017	0.6822	0.6693	0.6591	0.6503	0.6442	0.6389	0.6342	0.6311	0.6282	0.6234	0.6210	0.6187	0.6168	0.6154
13.50	0.7316	0.7121	0.6991	0.6887	0.6798	0.6737	0.6683	0.6636	0.6604	0.6575	0.6527	0.6502	0.6479	0.6460	0.6445
14.00	0.7615	0.7419	0.7288	0.7183	0.7093	0.7031	0.6977	0.6929	0.6897	0.6867	0.6818	0.6793	0.6770	0.6750	0.6736
14.50	0.7913	0.7717	0.7585	0.7479	0.7388	0.7325	0.7270	0.7222	0.7189	0.7158	0.7108	0.7062	0.7059	0.7039	0.7024
15.00	0.8212	0.8014	0.7881	0.7774	0.7682	0.7619	0.7562	0.7513	0.7479	0.7448	0.7397	0.7370	0.7346	0.7326	0.7311
15.50	0.8510	0.8312	0.8178	0.8069	0.7976	0.7911	0.7854	0.7804	0.7769	0.7737	0.7684	0.7656	0.7632	0.7612	0.7597
16.00	0.8809	0.8609	0.8474	0.8363	0.8269	0.8203	0.8145	0.8094	0.8058	0.8025	0.7969	0.7941	0.7916	0.7896	0.7880
16.50	0.9107	0.8907	0.8770	0.8658	0.8562	0.8495	0.8435	0.8363	0.8345	0.8311	0.8254	0.8224	0.8198	0.8178	0.8162
17.00	0.9406	0.9204	0.9066	0.8951	0.8854	0.8786	0.8724	0.8671	0.8632	0.8597	0.8537	0.8505	0.8479	0.8458	0.8442
17.50	0.9704	0.9501	0.9362	0.9245	0.9146	0.9076	0.9013	0.8958	0.8918	0.8881	0.8818	0.8765	0.8758	0.8737	0.8721
18.00	1.0002	0.9798	0.9657	0.9538	0.9437	0.9366	0.9300	0.9245	0.9203	0.9164	0.9098	0.9064	0.9036	0.9014	0.8997
18.50	1.0300	1.0095	0.9953	0.9831	0.9728	0.9656	0.9588	0.9530	0.9486	0.9446	0.9377	0.9340	0.9311	0.9289	0.9272
19.00	1.0599	1.0392	1.0248	1.0123	1.0018	0.9945	0.9874	0.9815	0.9769	0.9727	0.9654	0.9615	0.9586	0.9563	0.9545
19.50	1.0897	1.0689	1.0543	1.0415	1.0308	1.0233	1.0160	1.0099	1.0051	1.0007	0.9930	0.9889	0.9858	0.9835	0.9817
20.00	1.1195	1.0985	1.0837	1.0707	1.0597	1.0521	1.0445	1.0383	1.0332	1.0286	1.0204	1.0161	1.0129	1.0105	1.0087
20.50	1.1493	1.1282	1.1132	1.0999	1.0886	1.0808	1.0730	1.0665	1.0612	1.0564	1.0478	1.0432	1.0398	1.0374	1.0355
21.00	1.1791	1.1578	1.1426	1.1290	1.1175	1.1095	1.1014	1.0947	1.0892	1.0841	1.0749	1.0701	1.0666	1.0641	1.0622
21.50	1.2089	1.1874	1.1721	1.1581	1.1463	1.1381	1.1297	1.1229	1.1170	1.1116	1.1020	1.0968	1.0933	1.0907	1.0887
22.00	1.2387	1.2170	1.2015	1.1871	1.1751	1.1667	1.1580	1.1509	1.1448	1.1391	1.1289	1.1235	1.1198	1.1171	1.1151
22.50	1.2685	1.2466	1.2309	1.2162	1.2039	1.1953	1.1862	1.1789	1.1724	1.1665	1.1558	1.1500	1.1461	1.1434	1.1413
23.00	1.2982	1.2762	1.2602	1.2452	1.2326	1.2238	1.2144	1.2069	1.2000	1.1938	1.1825	1.1763	1.1723	1.1695	1.1674
23.50	1.3280	1.3058	1.2896	1.2742	1.2613	1.2522	1.2425	1.2347	1.2276	1.2210	1.2090	1.2026	1.1984	1.1955	1.1933
24.00	1.3578	1.3354	1.3190	1.3031	1.2899	1.2807	1.2706	1.2625	1.2550	1.2482	1.2355	1.2287	1.2243	1.2214	1.2191
24.50	1.3876	1.3650	1.3483	1.3321	1.3185	1.3090	1.2986	1.2903	1.2824	1.2752	1.2619	1.2546	1.2501	1.2471	1.2447
25.00	1.4173	1.3946	1.3776	1.3610	1.3471	1.3374	1.3265	1.3180	1.3097	1.3022	1.2881	1.2805	1.2758	1.2727	1.2702
25.50	1.4471	1.4241	1.4069	1.3899	1.3757	1.3657	1.3544	1.3456	1.3369	1.3290	1.3142	1.3062	1.3013	1.2981	1.2956
26.00	1.4769	1.4537	1.4362	1.4187	1.4042	1.3940	1.3823	1.3732	1.3641	1.3658	1.3403	1.3318	1.3267	1.3235	1.3209
26.50	1.5066	1.4832	1.4655	1.4476	1.4327	1.4222	1.4101	1.4007	1.3912	1.3825	1.3662	1.3573	1.3520	1.3487	1.3460
27.00	1.5364	1.5127	1.4948	1.4764	1.4611	1.4504	1.4379	1.4282	1.4182	1.4092	1.3920	1.3827	1.3772	1.3738	1.3710
27.50	1.5661	1.5423	1.5240	1.5052	1.4895	1.4786	1.4656	1.4556	1.4452	1.4357	1.4178	1.4079	1.4023	1.3987	1.3959
28.00	1.5959	1.5718	1.5533	1.5340	1.5179	1.5067	1.4933	1.4829	1.4721	1.4622	1.4434	1.4331	1.4272	1.4235	1.4206
28.50	1.6526	1.6013	1.5825	1.5627	1.5463	1.5348	1.5209	1.5102	1.4989	1.4886	1.4690	1.4581	1.4520	1.4483	1.4452
29.00	1.6554	1.6308	1.6117	1.5915	1.5747	1.5629	1.5485	1.5375	1.5257	1.5150	1.4944	1.4831	1.4768	1.4729	1.4698
29.50	1.6851	1.6603	1.6410	1.6202	1.6030	1.5909	1.5761	1.5647	1.5524	1.5412	1.5198	1.5079	1.5014	1.4974	1.4942
30.00	1.7148	1.6898	1.6702	1.6489	1.6313	1.6189	1.6036	1.5919	1.5791	1.5675	1.5450	1.5327	1.5259	1.5218	1.5165

The integral function on the left side of Eq. 34 can be evaluated by use of Table 34.2 from Ref. 8. These tables were prepared by using an arbitrary reference point of p_{pr} of 0.2. Evaluation of the integral is based on the following relationships:

$$\int_{(p_{pr})_2}^{(p_{pr})_1} \frac{(z/p_{pr})dp_{pr}}{1+B(z/p_{pr})^2} = \left[\int_{0.2}^{(p_{pr})_1} \frac{(z/p_{pr})dp_{pr}}{1+B(z/p_{pr})^2}\right]$$

$$-\left[\int_{0.2}^{(p_{pr})_2} \frac{(z/p_{pr})dp_{pr}}{1+B(z/p_{pr})^2}\right] = \frac{0.01877\gamma_g L}{\bar{T}}. \quad \ldots .(35)$$

Since the tables and charts provide numerical values for the bracketed terms in Eq. 35, *a calculation of flowing BHP can be obtained directly, with only simple mathematics being involved.*

In the previous and subsequent calculation procedures, the diameter of the flow string enters into the calculations as the fifth power. It is important, therefore, that the exact dimensions of the flow string be used rather than nominal flow-string sizes. Table 34.3 lists the pertinent information on various flow-string sizes.

The effect of assuming a constant average temperature over the entire gas column in Eqs. 17, 21, and 35 can be mitigated by taking only small increments of depth from top to bottom and using a constant temperature for each increment of depth. Assuming a linear temperature gradient, the average temperature for each depth increment can be calculated. The larger the number of depth increments taken in calculating the pressure traverse, the closer one approximates the rigorous integration of the equations.

Example Problem 3.[6] Calculate the BHP of a flowing-gas well. Given:

length of vertical pipe, L = 10,000 ft,
tubing ID, d_{ti} = 2.00 in.,
gas-flow rate, q_g = 4.91×10^6 cu ft/D,
flowing wellhead pressure, p_2 = 1,980 psia,
average flowing temperature, $\bar{T}$ = 636°R,
gas gravity (air=1.0), γ_g = 0.750,
p_{pc} = 660 psia,
T_{pc} = 400°R, and
f = 0.016.

Solution.

1. Calculate B.

$$B=\frac{667fq_g{}^2\bar{T}^2}{d_{ti}{}^5p_{pc}{}^2}=\frac{(667)(0.016)(4.91)^2(636)^2}{(2.00)^5(660)^2}=7.48.$$

2. Calculate $\dfrac{0.01877\gamma_g L}{\bar{T}}$.

$$\frac{0.01877\gamma_g L}{\bar{T}}=\frac{(0.01877)(0.750)(10{,}000)}{636}=0.2213.$$

TABLE 34.3—FLOW STRING WEIGHTS AND SIZES

Nominal Size (in.)	API Rating (in.)	Weight per Foot (lbm/ft)	OD (in.)	ID (in.)
1 3/4	....	2.3 or 2.4	1.660	1.380
1 1/2	....	2.9 or 2.748	1.900	1.610
2	2 3/8	4.00	2.375	2.041
2	2 3/8	4.5 or 4.7	2.375	1.995
2 1/2	2 7/8	5.897	2.875	2.469
2 1/2	2 7/8	6.25 or 6.5	2.875	2.441
3	3 1/2	7.694	3.500	3.068
3	3 1/2	8.50	3.500	3.018
3	3 1/2	9.30	3.500	2.992
3	3 1/2	10.2	3.500	2.922
3 1/2	4	9.26 or 9.50	4.000	3.548
3 1/2	4	11.00	4.000	3.476
4	4 1/2	10.98	4.500	4.026
4	4 1/2	11.75	4.500	3.990
4	4 1/2	12.75	4.500	3.958
4 1/2	4 3/4	16.00	4.750	4.082
4 1/2	4 3/4	16.50	4.750	4.070
4 3/4	5	12.85	5.000	4.500
4 3/4	5	13.00	5.000	4.494
4 3/4	5	15.00	5.000	4.408
4 3/4	5	18.00	5.000	4.276
4 3/4	5	21.00	5.000	4.154
.....	5 1/4	16.00	5.250	4.648
5 3/16	5 1/2	17.00	5.500	4.892
5 3/16	5 1/2	20.00	5.500	4.778
.....	5 3/4	14.00	5.750	5.290
.....	5 3/4	17.00	5.750	5.190
.....	5 3/4	19.50	5.750	5.090
.....	5 3/4	22.50	5.750	4.990
5 5/8	6	20.00	6.000	5.350
6 1/4	6 5/8	20.00	6.625	6.049
6 1/4	6 5/8	24.00	6.625	5.921
6 1/4	6 5/8	26.00	6.625	5.855
6 1/4	6 5/8	28.00	6.625	5.791
6 1/4	6 5/8	29.00	6.625	5.761
6 5/8	7	20.00	7.000	6.456
6 5/8	7	22.00	7.000	6.398
6 5/8	7	24.00	7.000	6.336
6 5/8	7	26.00	7.000	6.276
6 5/8	7	28.00	7.000	6.214
6 5/8	7	30.00	7.000	6.154
API	7 5/8	34.00	7.625	6.765
7 5/8	8	26.00	8.000	7.386
API	8 1/8	28.00	8.125	7.485
API	8 1/8	32.00	8.125	7.385
API	8 1/8	35.50	8.125	7.285
API	8 1/8	39.5 or 40.00	8.125	7.185
API	8 1/8	42.00	8.125	7.125
8 1/4	8 5/8	24.00	8.625	8.097
8 1/4	8 5/8	28.00	8.625	8.017
8 1/4	8 5/8	32.00	8.625	7.921
8 1/4	8 5/8	32.00	8.625	7.907
8 1/4	8 5/8	36.00	8.625	7.825
8 1/4	8 5/8	38.00	8.625	7.775
8 1/4	8 5/8	43.00	8.625	7.651
8 3/4	8 5/8	44.85	8.625	7.625
8 5/8	9	34.00	9.000	8.290
8 5/8	9	38.00	9.000	8.196
8 5/8	9	40.00	9.000	8.150
8 5/8	9	45.00	9.000	8.032
8 5/8	9	54.00	9.000	7.812
API	9 5/8	43.80	9.625	8.755
API	9 5/8	47.20	9.625	8.681
API	9 5/8	53.60	9.625	8.535
API	9 5/8	57.40	9.625	8.451
9 1/4	9 5/8	36.00	9.625	8.921
9 5/8	10	33.00	10.000	9.384
9 5/8	10	60.00	10.000	8.780
10	10 3/4	32.75	10.750	10.192
10	10 3/4	35.75	10.750	10.136
10	10 3/4	40.00	10.750	10.054
API	10 3/4	40.50	10.750	10.050
10	10 3/4	45.00	10.750	9.960
API	10 3/4	45.50	10.750	9.950
10	10 3/4	48.00	10.750	9.902
API	10 3/4	51.00	10.750	9.850
10	10 3/4	54.00	10.750	9.784

3. Calculate pseudoreduced wellhead pressure and pseudoreduced average temperature,

$$(p_{pr})_2=\frac{1,980}{660}=3.0$$

and

$$\bar{T}_{pr}=\frac{636}{400}=1.59.$$

4. For $\bar{T}_{pr}=1.59$, read from Table 34.2.

$$\int_{0.2}^{(p_{pr})_2}\frac{(z/p_{pr})dp_{pr}}{1+B(z/p_{pr})^2}=0.4246.$$

5. Add $\frac{0.01877\gamma_g L}{\bar{T}}$ to $\int_{0.2}^{(p_{pr})_2}\frac{(z/p_{pr})dp_{pr}}{1+B(z/p_{pr})^2}$.

$$0.4246+0.2213=0.6459.$$

6. From Table 34.2 find the pseudoreduced pressure corresponding to

$$\int_{0.2}^{(p_{pr})_1}\frac{(z/p_{pr})dp_{pr}}{1+B(z^2/p_{pr}{}^2)}=0.6459.$$

$$(p_{pr})_1=4.358.$$

7. Multiply $(p_{pr})_1$ by p_{pc} to obtain BHP.

$$p_1=4.358\times 660=2,876 \text{ psia}.$$

Another procedure for calculating the BHP of flowing gas wells that has found widespread use since its adoption by various state regulatory agencies is that of Cullender and Smith.[7] The method avoids the assumption of a constant average temperature by including the temperature within the integral.

$$18.75\gamma_g L=\int_{p_2}^{p_1}\frac{[p/(Tz)]dp}{F^2+0.001[p/(Tz)]^2}, \quad \ldots\ldots\ldots (36)$$

where

$$F^2=(2.6665f_f q_g{}^2)/d_i{}^5, \quad \ldots\ldots\ldots (37)$$

f_f is the Fanning friction factor and is equal to $f_f=f/4$, and f is the Moody friction factor from Fig. 34.2

Eq. 37 can be simplified by using the Nikuradse friction factor equation for fully turbulent flow and for an absolute roughness of 0.0006 in.:

$$F=F_r q_g=\frac{0.10797q_g}{d_i{}^{2.612}}, \quad \ldots\ldots\ldots (38)$$

where $d_i<4.277$ in. and

$$F=F_r q_g=\frac{0.10337q_g}{d_i{}^{2.582}}, \quad \ldots\ldots\ldots (39)$$

where $d_i>4.277$ in. Values of F_r are presented in Table 34.4 for various tubing and casing sizes.[7]

The right side of Eq. 36 may be integrated numerically by employing a two-step trapezoidal integration:

$$18.75\gamma_g L=\frac{(p_m-p_2)(I_m+I_2)}{2}+\frac{(p_1-p_m)(I_1+I_m)}{2}, \quad \ldots\ldots\ldots (40)$$

where

$$I=\frac{p/(Tz)}{F^2+0.001[p/(Tz)]^2}$$

and

$$p_m=p_1+(p_1+p_2)/2.$$

Eq. 40 may be separated into two expressions, one for each half of the flow string.

$$18.75\gamma_g L=(p_m-p_2)(I_m+I_2) \quad \ldots\ldots\ldots (41)$$

for the upper half, and

$$18.75\gamma_g L=(p_1-p_m)(I_1+I_m) \quad \ldots\ldots\ldots (42)$$

for the lower half.

By trial and error, p_m is calculated from Eq. 41. p_1 then is calculated in a similar manner by using the value of I_m from Eq. 41 and substituting in Eq. 42.

Simpson's rule then is employed to obtain a more accurate value of the BHP.

$$18.75\gamma_g L=\left(\frac{p_2-p_1}{6}\right)(I_2+4I_m+I_1). \quad \ldots\ldots\ldots (43)$$

Rather than using the two-step trapezoidal integration to make the first estimate of the BHP, Simpson's rule may be used directly and the BHP calculated by trial and error.

As this indicates, the Cullender and Smith method involves tedious trial and error solution if hand calculated. The method is best solved by computer. Quoting Ref. 8,

> Because the Cullender and Smith method considers both temperature and Z to be functions of pressure, it might appear that this method is somewhat more accurate than the Sukkar-Cornell approach. This is only an apparent advantage. If temperature is known in the gas column, it is possible to break the depth into several increments, each with one appropriate mean temperature.

This was alluded to previously. The Sukkar-Cornell method is an accurate, fast hand calculation procedure that avoids trial and error calculations. It is also amenable to computer solution.

Example Problem 4.[9] Calculate the flowing BHP by the method of Cullender and Smith from the following well data:

$$\begin{aligned} \text{gas gravity, } \gamma_g &= 0.75, \\ \text{length of vertical pipe, } L &= 10{,}000 \text{ ft}, \\ \text{wellhead temperature, } T_2 &= 570°\text{R}, \\ \text{formation temperature, } T_1 &= 705°\text{R}, \\ \text{wellhead pressure, } p_2 &= 2{,}000 \text{ psig}, \\ \text{flowrate, } q_g &= 4.915\times10^6 \text{ cu ft/D}, \\ \text{tubing ID, } d_{ti} &= 2.441 \text{ in.}, \\ \text{pseudocritical temperature, } T_{pc} &= 408°\text{R, and} \\ \text{pseudocritical pressure, } p_{pc} &= 667 \text{ psi.} \end{aligned}$$

$$\bar{T}=\frac{T_1+T_2}{2}=\frac{570+705}{2}=638°\text{R},$$

$$\text{wellhead } T_{pr}=\frac{T_2}{T_{pc}}=\frac{570}{408}=1.397,$$

$$\text{midpoint } T_{pr}=\frac{\bar{T}}{T_{pc}}=\frac{638}{408}=1.564,$$

$$\text{bottom } T_{pr}=\frac{T_i}{T_{pc}}=\frac{705}{408}=1.728,$$

$$\text{wellhead } p_{pr}=\frac{p_2}{p_{pc}}=\frac{2{,}000}{667}=2.999,$$

$$F=\frac{(0.10797)(4.915)}{(2.441)^{2.612}}=0.05158,$$

and $F^2=0.00266$.

Left side of Eq. 36,

$$18.75\ \gamma_g L=(18.75)(0.75)(10{,}000)$$

$$=140{,}625.$$

Calculate I_2. From the compressibility factor chart (see Chap. 20) $z_2=0.705$. Therefore,

$$\frac{p_2}{T_2 z_2}=\frac{2{,}000}{(570)(0.705)}=4.977$$

and

$$I_2=\frac{4.977}{0.00266+0.001(4.977)^2}=181.44.$$

Assume $I_2=I_m$. Solving Eq. 41 for p_m,

$$140{,}625=(p_m-2{,}000)(181.44+181.44),$$

$$p_m=2{,}388 \text{ psia}.$$

TABLE 34.4—VALUES OF F_r FOR VARIOUS TUBING AND CASING SIZES

$F_r=\dfrac{0.10797}{d^{2.612}}$ *

Nominal Size (in.)	OD (in.)	lbm/ft	ID (in.)	F_r
1	1.315	1.80	1.049	0.095288
1¼	1.660	2.40	1.380	0.046552
1½	1.990	2.75	1.610	0.031122
2	2.375	4.70	1.995	0.017777
2½	2.875	6.50	2.441	0.010495
3	3.500	9.30	2.992	0.006167
3½	4.000	11.00	3.476	0.004169
4	4.500	12.70	3.958	0.002970
4½	4.750	16.25	4.082	0.002740
	4.750	18.00	4.000	0.002889
4¾	5.000	18.00	4.276	0.002427
	5.000	21.00	4.154	0.002617

$F_r=\dfrac{0.01337}{d_i^{2.582}}$ **

Nominal Size (in.)	OD (in.)	lbm/ft	ID (in.)	F_r
4¾	5.000	13.00	4.494	0.0021345
	5.000	15.00	4.408	0.0022437
5 3/16	5.500	14.00	5.012	0.0016105
	5.500	15.00	4.976	0.0016408
	5.500	17.00	4.892	0.0017145
	5.500	20.00	4.778	0.0018221
	5.500	23.00	4.670	0.0019329
	5.500	25.00	4.580	0.0020325
5⅝	6.000	15.00	5.524	0.0012528
	6.000	17.00	5.450	0.0012972
	6.000	20.00	5.352	0.0013595
	6.000	23.00	5.240	0.0014358
	6.000	26.00	5.140	0.0015090
6¼	6.625	20.00	6.049	0.0009910
	6.625	22.00	5.989	0.0010169
	6.625	24.00	5.921	0.0010473
	6.625	26.00	5.855	0.0010781
	6.625	28.00	5.791	0.0011091
	6.625	31.80	5.675	0.0011686
	6.625	34.00	5.595	0.0012122
6⅝	7.000	20.00	6.456	0.0008876
	7.000	22.00	6.398	0.0008574
	7.000	24.00	6.336	0.0008792
	7.000	26.00	6.276	0.0009011
	7.000	28.00	6.214	0.0009245
	7.000	30.00	6.154	0.0009479
	7.000	40.00	5.836	0.0010871
7¼	7.625	26.40	6.969	0.0006875
	7.625	29.70	6.875	0.0007121
	7.625	33.70	6.765	0.0007424
	7.625	38.70	6.625	0.0007836
	7.625	45.00	6.445	0.0008413
	8.000	26.00	7.386	0.0005917
7⅝	8.125	28.00	7.485	0.0005717
	8.125	32.00	7.385	0.0005919
	8.125	35.50	7.285	0.0006132
	8.125	39.50	7.185	0.0006354
8¼	8.625	17.50	8.249	0.0004448
	8.625	20.00	8.191	0.0004530
	8.625	24.00	8.097	0.0004667
	8.625	28.00	8.003	0.0004810
	8.625	32.00	7.907	0.0004962
	8.625	36.00	7.825	0.0005098
	8.625	38.00	7.775	0.0005183
	8.625	43.00	7.651	0.0005403
8⅝	9.000	34.00	8.290	0.0004392
	9.000	38.00	8.196	0.0004523
	9.000	40.00	8.150	0.0004589
	9.000	45.00	8.032	0.0004765
9	9.625	36.00	8.921	0.0003634
	9.625	40.00	8.835	0.0003726
	9.625	43.50	8.755	0.0003814
	9.625	47.00	8.681	0.0003899
	9.625	53.50	8.535	0.0004074
	9.625	58.00	8.435	0.0004200
9⅝	10.000	33.00	9.384	0.0004167
	10.000	55.50	8.908	0.0003648
	10.000	61.20	8.790	0.0003775
10	10.750	32.75	10.192	0.0002576
	10.750	35.75	10.136	0.0002613
	10.750	40.00	10.050	0.0002671
	10.750	45.50	9.950	0.0002741
	10.750	48.00	9.902	0.0002776
	10.750	54.00	9.784	0.0002863

*Use only for internal diameters less than 4.277 in.
**Use only for internal diameters greater than 4.277 in.

Second trial:

$$p_{pr}=\frac{p_m}{p_{pc}}=\frac{2,388}{667}=3.580,$$

$z_m=0.800$ at $T_{pr}=1.564$, $p_{pr}=3.580$,

$$\frac{p_m}{T_m z_m}=\frac{2,388}{(638)(0.800)}=4.679,$$

and

$$I_m=\frac{4.679}{(0.00266)+0.001(4.679)^2}=190.57.$$

Solving Eq. 41 for p_m,

$$140,625=(p_m-2,000)(190.57+181.44)$$

and

$p_m=2,378$ psia.

Third trial:

$$p_{pr}=\frac{p_m}{p_{pc}}=\frac{2,378}{667}=3.565,$$

$z_m=0.800$ at $T_{pr}=1.564$, $p_{pr}=3,565$,

$$\frac{p_m}{T_m z_m}=\frac{2,378}{(638)(0.800)}=4.659,$$

and

$$I_m=\frac{4.659}{0.00266+0.001(4.659)^2}=191.21.$$

Solving Eq. 41 for p_m,

$$140,625=(p_m-2,000)(191.21+181.44),$$

therefore

$p_m=2,377$ psia.

For the lower half of the flow string assume $I_1=I_m=191.21$. Solving Eq. 42 for p_1,

$$140,625=(p_1-2,377)(191.21+191.21),$$

$p_1=2,745$ psia.

Second trial:

$$p_{pr}=\frac{p_1}{p_{pc}}=\frac{2,745}{667}=4.115,$$

$z_1=0.869$ at $T_{pr}=1.728$, $p_{pr}=4.115$,

$$\frac{p_1}{T_1 z_1}=\frac{2,745}{(705)(0.869)}=4.481$$

and

$$I_1=\frac{4.481}{0.00266+0.001(4.481)^2}=197.06.$$

Solving Eq. 42 for p_1,

$$140,625=(p_1-2,377)(197.06+191.21),$$

$p_1=2,739$ psia.

Third trial:

$$p_{pr}=\frac{p_1}{p_{pc}}=\frac{2,739}{667}=4.106,$$

$z_1=0.869$ at $T_{pr}=1.728$, $p_{pr}=4.106$,

$$\frac{p_1}{T_1 z_1}=\frac{2,739}{(705)(0.869)}=4.471,$$

and

$$I_1=\frac{4.471}{0.00266+0.001(4.471)^2}=197.40.$$

Solve Eq. 42 for p_1.

$$140,625=(p_1-2,377)(197.40+191.21),$$

$p_1=2,739$ psia.

Using Simpson's rule from Eq. 43,

$$140,625=\frac{(p_1-p_2)}{6}\times[181.44+4(191.21)+197.40],$$

$p_1-p_2=738,$

and

$p_1=738+2,000=2,738$ psia.

A simplified method for calculating flowing BHP of gas wells results if an effective average temperature and an effective average compressibility are used over the length of the flow string. Low-pressure wells at shallow depths or wells where pressure drop is small are especially well suited for this method. With the usual assumptions that kinetic energy is negligible, g/g_c equals unity, etc., the following equation for vertical gas flow has been developed by Smith[10]:

$$p_{bh}^2-e^s p_{th}^2=\frac{25 f q_g^{\,2}\bar{T}^2\bar{z}^2(e^s-1)}{0.0375 d_i^{\,5}}, \quad \ldots\ldots\ldots (44)$$

where
p_{bh} = BHP, psia,
p_{th} = tophole pressure, psia,
f = friction factor, dimensionless, from Fig. 34.2,
q_g = gas flow rate, 10^6 cu ft/D referred to 14.65 psia and 60°F,
s = exponent of $e = \frac{0.0375\gamma_g L}{\bar{T}\bar{z}}$,
γ_g = gas gravity (air=1.0),
L = length of vertical flow string, ft,
$\bar{T}$ = average temperature, °R,
$\bar{z}$ = average compressibility of gas, dimensionless,
d_i = internal diameter of flow string, in., and
e = natural logarithm base=2.71828.

The method using Eq. 44 is also a trial and error procedure.

In evaluating the friction factor for commercial pipe, Smith[10] and Cullender and Binckley[11] have shown from an analysis of flow data that average absolute values of roughness, 0.00065 and 0.0006 in., respectively, are the correct values to use for clean commercial pipe. For an absolute roughness of 0.0006 in., Cullender and Binckley[11] derived an expression for the friction factor as defined in Fig. 34.2, as a power function of the Reynolds number and pipe diameter. In terms of field units,

$$f=30.9208\times 10^{-3}\frac{q_g^{-0.065}d_i^{-0.058}\gamma_g^{-0.065}}{\mu_g^{-0.065}}, \quad \ldots\ldots (45)$$

where
q_g = gas flow rate, 10^6 cu ft/D,
d_i = internal diameter of flow string, ft,
γ_g = gas gravity (air=1.0), and
μ_g = gas viscosity, lbm/ft-sec.

Flow Through a Tubing-Casing Annulus. The flow equations that relate to flow through a circular pipe, when properly modified, can be used for conditions where flow is through an annular space. This modification involves determining the hydraulic radius of the annular cross section and using the friction factor obtained for an "equivalent" (i.e., having the same hydraulic radius) circular pipe. The hydraulic radius is defined as the area of flow cross section divided by the wetted perimeter. For a circular pipe,

$$r_H=\frac{\pi d_i^2/4}{\pi d_i}=\frac{d_i}{4}. \quad \ldots\ldots (46)$$

For a tubing-casing annulus,

$$r_H=\frac{(\pi/4)(d_{ci}^2-d_{to}^2)}{\pi(d_{ci}+d_{to})}=\frac{d_{ci}-d_{to}}{4}, \quad \ldots\ldots (47)$$

where
d_{ci} = inside diameter of casing, ft,
d_{to} = outside diameter of tubing, ft, and
r_H = hydraulic radius, ft.

The diameter of an equivalent circular pipe, thus, would be

$$d_{eq}=d_{ci}-d_{to}. \quad \ldots\ldots (48)$$

Modification of Eq. 32 for annular flow involves only substituting d_{eq} for d_i. Likewise d_{eq} replaces d_i when determining friction factor (from the Reynolds-number plot, Fig. 34.2). However, the simplification of Eq. 32 includes velocity expressed as a function of diameter and volumetric flow rate, and so d_i^5 in B of Eq. 33 and in Eq. 44 becomes

$$d_i^5=(d_{ci}+d_{to})^2(d_{ci}-d_{to})^3. \quad \ldots\ldots (49)$$

Gas/Water Flow

The effect of water production on calculated pressure drop for gas wells operating in mist flow can be included by using an average density assuming zero slip velocity and by using total rate in the friction loss term. The volumetric average density can be calculated as

$$\bar{\rho}=\frac{q_g\rho_g+q_w\rho_w}{q_g+q_w}$$

where $\bar{\rho}$ is the average density at flowing conditions and q is the volumetric flow rate at flowing conditions. To include the effect of water in the Cullender and Smith calculation, modify the integrand, I, as follows (see Page 24):

$$I=\frac{[p/(Tz)](\bar{\rho}/\rho_g)}{F^2\left(\frac{q_g+q_w}{q_g}\right)^2+0.001[p/(Tz)]^2(\bar{\rho}/\rho_g)^2}.$$

Gas-Condensate Wells

Calculation of BHP. Calculations of BHP on gas-condensate wells are based on equations previously presented for gas wells. The application of these equations may be limited somewhat by the amount of liquid present in the flow string.

Upon shutting in a gas-condensate well, part of the liquids that were being carried in the flow stream may fall back and accumulate in the bottom of the wellbore. For this reason, it is advisable to determine whether or not such a static liquid level exists in a gas-condensate well before relying on a BHP calculated from surface measurements. When the location of the static liquid level is known, the gas calculations can be used to determine the pressure at the gas-liquid interface and the length of the liquid column. An estimated liquid density will provide the additional pressure needed to determine pressure at formation level.

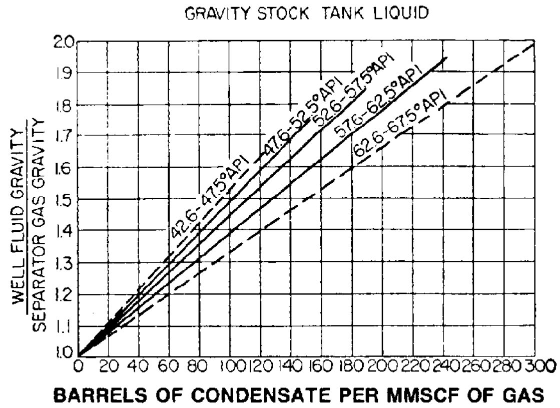

Fig. 34.4—Gas/gravity ratio vs. condensate/gas ratio as a function of condensate gravity.

In the flow equations for gas, the gas gravity is the flow-stream gravity. This is calculated for condensates from the following[12]:

$$\gamma_g = \frac{(\gamma_g)_{sp} + (4{,}591\gamma_L/R_{gL})}{1+(1.123/R_{gL})}, \quad \text{(50)}$$

where

$(\gamma_g)_{sp}$ = separator gas gravity (air=1),
γ_L = specific gravity of condensate, and
R_{gL} = gas-liquid ratio, cu ft/bbl.

Nisle and Poettmann[13] published a simple correlation based on field data (Fig. 34.4) that can be used to calculate the flow-stream gravity of the entrained mixture such as occurs in the case of a flowing gas-condensate well.

Accuracy of the flow equations for gas, as modified for gas-condensate wells, is influenced by the amount of liquid in the flow stream. The higher the gas-liquid ratio, the more accurate the calculated results will be.

Injection Wells

Petroleum-production operations often involve the injection of fluids into the subsurface formation, as is the case in waterflooding, pressure maintenance, gas cycling, and designing gas lift installations. Therefore, it becomes desirable to have a means of predicting the variation of pressure with depth for the vertical downward flow of fluids. Eqs. 29 and 30, previously discussed, form the basis of any specific fluid-flow relationship. They contain no limiting assumptions other than those arrived at in deriving Eq. 30 from Eq. 29. The only difference in applying Eq. 30 to vertical downward flow when compared with upward flow is that the integration limits are changed; that is, the sign of the *absolute values* of potential energy then changes and, depending on the rate of injection in the case of gas injection, the absolute value of the compressional energy change may vary from positive to negative. In other words, at low flow rates, the BHP is greater than the surface pressure; whereas, at high flow rates, the BHP is less than the surface pressure.

Liquid Injection

Calculation of Injection BHP. For isothermal flow of incompressible fluid, assuming $g/g_c=1$, and integrating between limits of the top and bottom of the hole, Eq. 30 may be written as follows:

$$\frac{\Delta p}{\rho} + \Delta Z + E_\ell = 0. \quad \text{(51)}$$

(Since the datum plane is at the surface, ΔZ will be a negative number.) Then

$$p_2 = p_1 - \Delta Z\rho - E_\ell \rho, \quad \text{(52)}$$

since $-\Delta Z = D$, the depth. Therefore,

$$p_2 = p_1 + D\rho - E_\ell \rho. \quad \text{(53)}$$

Since $E_\ell = fv^2D/2g_c d_i$ (Fig. 34.2),

$$p_2 = p_1 + D\rho - \frac{fv^2 D\rho}{2g_c d_i}. \quad \text{(54)}$$

Converting pressure units to pounds per square inch,

$$p_2 = p_1 + \frac{D\rho}{144} - \frac{fv^2 D\rho}{288 g_c d_i}, \quad \text{(55)}$$

where

p_2 = bottomhole pressure, psia, at depth D,
p_1 = surface pressure, psia,
D = depth of well, ft,
ρ = density of injected fluid, lbm/cu ft,
f = friction factor (Fig. 34.2),
v = fluid velocity, ft/sec,
d_i = internal diameter of pipe, ft, and
g_c = 32.2 conversion factor.

Eq. 55 reveals that the BHP for the case of incompressible flow as assumed for liquid injection into a wellbore is simply the surface pressure plus the pressure from the "weight of the liquid column" minus the pressure drop caused by frictional effects. For no flow, it reduces to the well-known expression for a static-fluid column

$$p_2 = p_1 + \frac{D\rho}{144}. \quad \text{(56)}$$

Gas Injection

Calculation of Injection BHP. Starting with the general differential equation, Eq. 30, Poettmann[4] derived an expression for calculating the sandface pressure of flowing-gas wells in which the variation of the compressibility factor of the gas with pressure is taken into consideration. The same integral factor as given in Table 34.1 is employed for the calculation of static BHP in Table 34.5.

By following the same reasoning as in the previous section, the equation can be rearranged so that the pressure traverse for vertical flow downward can be calculated as follows:

$$D=\frac{D_s}{\{0.9521\times10^{-6}[fq_g{}^2\gamma_g{}^2D_s{}^2/d_{ti}{}^5(\Delta p)^2]\}-1}, \quad \text{.......... (57)}$$

where

D = depth of well, ft,
Δp = p_2-p_1, psia,
d_{ti} = ID of tubing, ft,
q_g = gas flow, 10^6 cu ft/D at 14.65 psia and 60°F,
f = friction factor (Fig. 34.2), and
D_s = D under static conditions (static equivalent depth for pressures encountered at flowing conditions)

$$=\frac{53.241\bar{T}}{\gamma_g}\times\left[\int_{0.2}^{(p_{pr})_2}\frac{z}{p_{pr}}\mathrm{d}p_{pr}-\int_{0.2}^{(p_{pr})_1}\frac{z}{p_{pr}}\mathrm{d}p_{pr}\right].$$

Using the expression for the friction factor as derived by Cullender and Binckley[11] (Eq. 45) and substituting in Eq. 57 gives

$$D=\frac{D_s}{\{2.944\times10^{-8}[q_g{}^{1.935}\gamma_g{}^{1.935}D_s{}^2/(d_{ti}{}^{5.058}\mu^{0.065})(\Delta p)^2]\}-1} \quad \text{.......... (58)}$$

Cullender and Smith's Eq. 36 also can be rearranged to calculate the BHP for the case of gas injection as follows:

$$18.75\gamma_gD=\int_{p_1}^{p_2}\frac{[p/(Tz)]\mathrm{d}p}{0.001\left(\dfrac{p}{Tz}\right)^2-F^2}. \quad \text{.......... (59)}$$

The solution of this equation is identical to that previously described for flowing gas wells. D, depth of well, can be used interchangeably with L, length of flow string, when the well is vertical.

Similarly, by considering the downward flow of gas, the simplified equation developed by Smith[10] for upward flow (Eq. 44) can be rearranged so that the pressure traverse for vertical flow downward can be calculated.

$$e^sp_{th}{}^2-p_{bh}{}^2=\frac{25fq_g{}^2\bar{T}^2z(e^s-1)}{0.0375d_i{}^5}. \quad \text{.......... (60)}$$

The nomenclature is the same as used in the corresponding Eq. 44.

In the case of gas injection down the annulus of a well, $d_{ti}{}^5$ of Eq. 57 (or $d_i{}^5$ of Eq. 60) is replaced as defined in Eq. 49; that is,

$$d_i{}^5=(d_{ci}+d_{to})^2(d_{ci}-d_{to})^3.$$

In the case of annulus injection using Eq. 58, $d_{ti}{}^{5.058}$ is replaced as follows:

$$d_i{}^{5.058}=(d_{ci}+d_{to})^{1.935}(d_{ci}-d_{to})^{3.123}. \quad \text{....... (61)}$$

Eqs. 57 through 60 provide a basis for calculating the BHP in a gas-injection well. In solving Eqs. 57 and 58, the calculating procedure is to assume a pressure p_2 and solve for the corresponding depth, D. The depth, D, so found will be the depth at which pressure p_2 occurs. By calculating several such points, a pressure-depth traverse can be plotted from which the pressure at the desired depth can be determined.

It is apparent that BHP during gas injection can be either greater or less than tophole pressure depending on the energy losses encountered. At low rates of flow, the pressure gradient is positive, whereas at high flow rates, the pressure gradient is negative. This is because, as flow rate increases, energy or frictional losses increase and they can be overcome only by a *decrease* in the *change of compression energy* or pV energy of the system. The decrease in potential energy resulting from elevation is constant and the change in kinetic energy is negligible. This can be illustrated by examining and rearranging Eq. 4 and considering the kinetic energy negligible.

$$\int_{p_1}^{p_2}V\mathrm{d}p+E_\ell=-\frac{g}{g_c}\Delta Z. \quad \text{.................. (62)}$$

For low flow rates,

$$\int_{p_1}^{p_2}V\mathrm{d}p$$

is positive and E_ℓ is always positive; thus, the sum of the compression energy and energy losses must equal the change in potential energy, which for a given depth is constant (the absolute value of $-\Delta Z$ is positive for gas injection since the absolute value of ΔZ is negative).

As E_ℓ increases with flow rate, the

$$\int_{p_1}^{p_2}V\mathrm{d}p$$

must decrease for the sum to remain constant. When E_ℓ is equal to $(g/g_c)\,\Delta Z$, the pressure at the top and bottom of the hole is the same. This means that the decrease in potential energy is equal to the frictional losses. As E_ℓ further increases, the added energy to overcome friction losses must come from the compressional energy since $-(g/g_c)\,\Delta Z$ is constant. This then means that the pressure gradient is negative.

TABLE 34.5—SAMPLE CALCULATIONS

Δp (1)	p (2)	p_{pr} (3)	$\int_{0.2}^{p_{pr}} \frac{z}{p_{pr}} dp_{pr}$ (4)	$\int_{(p_{pr})_1}^{(p_{pr})_2} \frac{z}{p_{pr}} dp_{pr}$ (5)	D_s (6)	D (7)	Depth corresponding to p of column (2) (8)
	680	1.015	1.586				0
20	700	1.045	1.611	0.025	1,278	−1,460	1,460
20	720	1.074	1.636	0.025	1,278	−1,460	2,920
20	740	1.104	1.662	0.026	1,329	−1,532	4,452

Example Problem 5. Calculate the pressure at 4,000 ft in a gas injection well. Given:

tubing ID, d_{ti} = 0.1663 ft,
gas flow rate, q_g = 0.783×10^6 cu ft/D,
average temperature, $\bar{T}$ = 600°R,
wellhead injection pressure, p_1 = 680 psia,
gas gravity, γ_g = 0.625, and
gas viscosity, μ_g = 8.74×10^{-6} lbm/ft-sec.

Solution.
1. Substitute given values in Eq. 58.

$$D=\frac{D_s}{\dfrac{2.944\times10^{-8}(0.783)^{1.935}(0.625)^{1.935}D_s^{\,2}}{(0.1663)^{5.058}(8.75\times10^{-6})^{-0.065}(\Delta p)^2}-1}$$

$$=\frac{D_s}{\dfrac{(3.00\times10^{-5})D_s^{\,2}}{(\Delta p)^2}-1}$$

and

$$D_s=\frac{53.241(600)}{0.625}\left[\int_{0.2}^{(p_{pr})_2}\frac{z}{p_{pr}}dp_{pr}-\int_{0.2}^{(p_{pr})_1}\frac{z}{p_{pr}}dp_{pr}\right]=51,100\left[\int_{0.2}^{(p_{pr})_2}\frac{z}{p_{pr}}dp_{pr}-\int_{0.2}^{(p_{pr})_1}\frac{z}{p_{pr}}dp_{pr}\right].$$

2. Determine p_{pc} and T_{pc} (Fig. 34.3).

$$p_{pc}=670 \text{ psia}$$

and

$$T_{pc}=365°\text{R},$$

therefore,

$$T_{pr}=\frac{\bar{T}}{T_{pc}}=\frac{600}{365}=1.64.$$

3. Assume values for Δp and solve for D (Table 34.5).
4. From plot of Col. 2 vs. Col. 8 read pressure at 4,000 ft to be 734 psia.

Oil Wells

Inflow Performance

The simplest and most widely used inflow performance or backpressure equation used to determine stabilized or pseudosteady-state flow at any backpressure p_{wf} is given by the productivity index (PI) equation as

$$q_o=J(\bar{p}_R-p_{wf}). \qquad (63)$$

In terms of measured data the PI is represented as

$$J=\frac{q_o}{\bar{p}_R-p_{wf}}, \qquad (64)$$

where
J = stabilized productivity index, STB/D-psi,
q_o = measured stabilized surface oil flow rate, STB/D,
p_{wf} = wellbore stabilized flowing pressure, psia, and
$\bar{p}_R$ = average reservoir pressure, psia.

J is defined specifically as a PI determined from flow rate and pressure drawdown measurements. It normally varies with increasing drawdown (i.e., is not a constant value). In terms of reservoir variables, the stabilized or pseudosteady-state PI J^* at zero drawdown or as $p_{wf}\rightarrow p_R$ can be written as

$$J^*=\frac{7.08kh}{\left[\ln\left(\dfrac{r_e}{r_w}\right)-\dfrac{3}{4}+s\right]}\left(\frac{k_{ro}}{\mu_o B_o}\right)_{p_R}, \qquad (65)$$

where
J^* = stabilized PI at zero drawdown, STB/D-psi,
k = effective permeability, darcy,
k_{ro} = relative permeability to oil, fraction,
h = formation thickness, ft,
μ_o = oil viscosity, cp (evaluated at $\bar{p}_R$),
B_o = oil formation volume factor, RB/STB (evaluated at $\bar{p}_R$),
r_e = external boundary radius, ft,
r_w = wellbore radius, ft, and
s = skin effect, dimensionless.

J^* is the special definition of PI J at a vanishing pressure drawdown (i.e., as p_{wf} approaches $\bar{p}_R$). PI for a well is defined uniquely only at a zero drawdown.

Although this discussion will be limited to the pseudosteady state, a transient form of the flow coefficient $J^*_{(t)}$ also is given for completeness.

$$J^*_{(t)} = \frac{7.08kh}{\left(\ln\sqrt{\dfrac{14.23kt}{\phi\mu c_t r_w^2}}+s\right)}\left(\frac{k_{ro}}{\mu_o B_o}\right)_{p_R}, \quad \ldots\ldots\ldots (66)$$

where t is time, days, ϕ is porosity, fraction, and c_t is total compressibility, psi^{-1}.

The above equations are perfectly valid for single-phase flow (i.e., $\bar{p}_R$ and p_{wf} are always greater than the reservoir bubblepoint pressure, p_b). However, it has long been recognized that in reservoirs existing at or below the bubblepoint pressure, producing wells do not follow the simple PI Eqs. 63 and 64. Actual field tests indicate that oil flow rates obtained at increasing drawdowns decline much faster than would be predicted by Eq. 63.

Evinger and Muskat[14] first derived a theoretical PI for steady-state radial flow in an attempt to account for the observed nonlinear flow behavior of oil wells. They arrived at the following equation:

$$q_o = \frac{7.08kh}{\ln\left(\dfrac{r_e}{r_w}\right)}\int_{p_{wf}}^{p_e} f(p)\mathrm{d}p, \quad \ldots\ldots\ldots (67)$$

where p_e is the reservoir pressure at the external boundary, psia, and

$$f(p) = \left(\frac{k_{ro}}{\mu_o B_o}\right).$$

Calculations using Eq. 67 with typical reservoir and fluid properties indicated that PI at a fixed reservoir pressure p_e decreases with increasing drawdown. This apparently complex form of an inflow-performance-relationship (IPR) equation found little use in the field.

In a computer study by Vogel,[15] results based on two-phase flow theory were presented to indicate that a single empirical IPR equation might be valid for most solution-gas-drive reservoirs. He found that a single dimensionless IPR equation approximately held for several hypothetical solution-gas drive reservoirs even when using a wide range of oil PVT properties and reservoir relative permeability curves. The fact that his study covered a wide range of fluid properties and relative permeability curves to obtain a single reference curve cannot be overemphasized. Vogel proposed that his simple equation be used in place of the linear PI relationship for solution-gas-drive reservoirs when the reservoir pressure is at or below the bubblepoint pressure.

The proposed equation (IPR) in dimensionless form was given as

$$\frac{q_o}{q_{o(\max)}} = 1-0.20\left(\frac{p_{wf}}{\bar{p}_R}\right)-0.80\left(\frac{p_{wf}}{\bar{p}_R}\right)^2, \quad \ldots\ldots (68)$$

where $q_{o(\max)}$ is the maximum producing rate at $p_{wf}=0$ psia.

Fetkovich,[16] in an attempt to verify the Vogel IPR relationship, obtained isochronal and flow-after-flow multipoint backpressure test field data on some 40 different oil wells. The reservoirs in which oilwell multipoint backpressure tests were obtained ranged from highly undersaturated, to saturated at initial reservoir pressure, to a partially depleted field with a gas saturation existing above the critical (equilibrium) gas saturation. A form of an IPR equation similar to that used for gas wells was found to be valid for tests conducted in all three reservoir fluid states, even for the conditions where flowing pressures were well above the bubblepoint pressures. Permeabilities of the reservoirs ranged from 6 to >1,000 md.

In all cases, oilwell backpressure curves were found to follow the same general form as that used to express the rate-pressure relationship of a gas well:

$$q_o = J'(\bar{p}_R^{\,2}-p_{wf}^{\,2})^n. \quad \ldots\ldots\ldots (69)$$

For the 40 oilwell backpressure tests examined, the exponent n was found to lie between 0.568 and 1.000—that is, within the limits commonly accepted for gas well backpressure curves.

In terms of measured data, J' is defined by

$$J' = \frac{q_o}{(\bar{p}_R^{\,2}-p_{wf}^{\,2})^n}, \quad \ldots\ldots\ldots (70)$$

where J' is the stabilized PI, STB/D (psi^2)^n. The exponent n usually is determined from a multipoint or isochronal backpressure test and is an indicator of the existence of non-Darcy flow. If $n=1$, non-Darcy flow is assumed not to exist.

With PI expressed in terms of pressures squared, $\bar{p}_R^{\,2}$ and $p_{wf}^{\,2}$,

$$J' = \frac{J^*}{2\bar{p}_R}. \quad \ldots\ldots\ldots (71)$$

Expressing the pseudosteady state J' in terms of reservoir variables,

$$J' = \frac{7.08kh}{2\bar{p}_R\left[\ln\left(\dfrac{r_e}{r_w}\right)-\dfrac{3}{4}+s\right]}\left(\frac{k_{ro}}{\mu_o B_o}\right)_{p_R}, \quad \ldots\ldots (72)$$

or

$$q_o = \frac{7.08kh}{\left[\ln\left(\dfrac{r_e}{r_w}\right)-\dfrac{3}{4}+s\right]}\left(\frac{k_{ro}}{\mu_o B_o}\right)_{p_R}$$

$$\cdot\frac{(\bar{p}_R^{\,2}-p_{wf}^{\,2})^{1.0}}{2\bar{p}_R}. \quad \ldots\ldots\ldots (73)$$

Expressed in a form with reservoir variables and a non-Darcy flow term, F_{Da}, where the resulting n would be less than 1.0 and a function of F_{Da},

$$q_o=\frac{7.08kh}{\left[\ln\left(\frac{r_e}{r_w}\right)-\frac{3}{4}+s+F_{Da}q_o\right]}\left(\frac{k_{ro}}{\mu_oB_o}\right)_{p_R}\cdot\frac{(\bar{p}_R^{\,2}-p_{wf}^{\,2})}{2\bar{p}_R}. \quad (74)$$

When $\bar{p}_R$ is equal to or less than the bubblepoint pressure p_b and n is less than 1, a non-Darcy flow factor, F_{Da}, is indicated. When $F_{Da}=0$, $n=1$. The term F_{Da} normally is developed from multipoint test data. As shown in a later example, it is possible to have $F_{Da}=0$ and n less than 1.0 for undersaturated wells producing at flowing pressures below the bubblepoint pressure. (See Fig. 8 of Ref. 16.) This is strictly a result of the shape of the $k_{ro}/(\mu_oB_o)$ pressure function.

Expressing the backpressure form of the IPR equation in terms similar to that of Vogel's equation (instead of Vogel's equation in terms of the backpressure curve), we have, from Eq. 69,

$$q_o=J'(p_R^{\,2}-p_{wf}^{\,2})^n$$

and

$$q_{o(\max)}=J'(p_R^{\,2})^n$$

or

$$J'=\frac{q_{o(\max)}}{(p_R^{\,2})^n}. \quad (75)$$

Substituting and rearranging yields

$$\frac{q_o}{q_{o(\max)}}=\left(\frac{p_R^{\,2}-p_{wf}^{\,2}}{p_R^{\,2}}\right)^n=\left[1-\left(\frac{p_{wf}}{p_R}\right)^2\right]^n. \quad (76)$$

For $n=1$, we have the simplest possible form of a multiphase IPR equation based on results obtained from actual field data:

$$\frac{q_o}{q_{o(\max)}}=1-\left(\frac{p_{wf}}{p_R}\right)^2. \quad (77)$$

Comparing Eq. 77 to Vogel's Eq. 68, which was derived only from computer simulation data, we see that the coefficient for $p_{wf}/\bar{p}_R$ is 0, and the coefficient for $(p_{wf}/p_R)^2$ is equal to 1. This results in an IPR Eq. 77 that yields a slightly more conservative answer than given by Vogel's original equation. (Actually, Vogel's Fig. 7 shows computer model calculated IPR results less than obtained from his reference equation.[15]) Not included in any of Vogel's simulation runs were effects of non-Darcy flow in the reservoir or perforation restrictions, which in the field result in n values less than 1.0 and an even more severe IPR rate reduction relationship.

Example Problem 6 (IPR). The following example illustrates the various possible methods of computing inflow rates.

An oil well is producing at a stabilized rate of 70 STB/D at a flowing BHP $p_{wf}=1{,}147$ psia. The average reservoir shut-in static pressure, $\bar{p}_R=1{,}200$ psia. Calculate the maximum possible flow rate, q_o, at 0 psig, and the producing rate if artificial lift were installed to lower the flowing BHP to 550 psia. Make the calculations using the PI Eq. 63, Vogel's method, and the backpressure curve method with $n=1.0$ and $n=0.650$. (The data are from an actual IPR test reported in Ref. 16.)

Productivity Index (PI)

$$J=\frac{70}{1{,}200-1{,}147}=1.32 \text{ STB/D-psi};$$

$$q_o \text{ (15 psi)}=J(\bar{p}_R-p_{wf})$$

$$=1.32\ (1{,}200-15)=1{,}564 \text{ STB/D};$$

$$q_o \text{ (550 psi)}=1.32(1{,}200-550)=858 \text{ STB/D}.$$

Vogel IPR

$$q_o=70 \text{ BOPD};\ \frac{p_{wf}}{p_R}=\frac{1{,}147}{1{,}200}=0.9558;$$

$$\left(\frac{p_{wf}}{\bar{p}_R}\right)^2=0.9136;$$

$$\frac{q_o}{q_{o(\max)}}=1-0.20\left(\frac{p_{wf}}{\bar{p}_R}\right)-0.80\left(\frac{p_{wf}}{p_R}\right)^2$$

$$=1-0.19116-0.73088=0.07796;$$

$$q_{o(\max)}=\frac{70}{0.07796}=898 \text{ BOPD};$$

and q_o at $p_{wf}=15$ psia.

$$\frac{q_o(15\text{ psi})}{q_{o(\max)}}=1-0.20\left(\frac{15}{1{,}200}\right)$$

$$-0.80\left(\frac{15}{1{,}200}\right)^2=0.99738;$$

$$q_o(15\text{ psi})=q_{o(\max)}(0.99738)$$

$$=898(0.99738)=896 \text{ BOPD};$$

q_o at $p_{wf}=550$ psia.

$$\frac{q_o(550\text{ psi})}{q_{o(\max)}}=1-0.20\left(\frac{550}{1{,}200}\right)$$

$$-0.80\left(\frac{550}{1,200}\right)^2=0.740277;$$

$q_o(550 \text{ psi})=q_{o(\max)}(0.740277)$

$=898(0.740277)=665$ BOPD.

Backpressure Curve ($n=1.0$) IPR

$q_o=70$ BOPD; $\bar{p}_R{}^2=(1,200)^2=1,440,000;$

$p_{wf}{}^2=(1,147)^2=1,315,609;$

$$J'=\frac{70}{(1,200)^2-(1,147)^2}$$

$$=\frac{70}{124,391}=0.00056274 \text{ STB/D-psi}^2;$$

$q_o(15 \text{ psi})=J'(p_R{}^2-p_{wf}{}^2)$

$=0.00056274\ (1,440,000-225)=810$ BOPD;

$q_o(550 \text{ psi})=0.00056274(1,440,000-302,500)$

$=640$ BOPD.

Using the dimensionless backpressure curve form in terms of $q_o/q_{o(\max)}$ and p_{wf}/p_R with $n=1.0$,

$$q_o=70 \text{ BOPD: } \left(\frac{p_{wf}}{p_R}\right)^2=\left(\frac{1,147}{1,200}\right)^2=0.9136;$$

$$\frac{q_o}{q_{o(\max)}}=1-\left(\frac{p_{wf}}{p_R}\right)^2=1-0.9136=0.0864;$$

$$q_{o(\max)}=\frac{70}{0.0864}=810 \text{ BOPD};$$

q_o at $p_{wf}=550$ psia.

$$\frac{q_o(550 \text{ psi})}{q_{o(\max)}}=1-\left(\frac{550}{1,200}\right)^2$$

$=1-0.168056=0.78993;$

$q_o(550 \text{ psi})=810(0.78993)=640$ BOPD.

Backpressure Equation ($n=0.650$) IPR

$q_o=70$ BOPD; $p_R=(1,200)^2=1,440,000;$

$p_{wf}=(1,147)^2=1,315,609;$

$$J'=\frac{70}{(1,440,000-1,315,609)^{0.650}}=\frac{70}{2,049.3}$$

$=0.0341580$ STB/D-psi^{2n};

$q_o\ (15 \text{ psi})=J'(\bar{p}_R{}^2-p_{wf}{}^2)^{0.650}$

$=0.0341580(1,440,000-225)^{0.650};$

$q_o\ (15 \text{ psi})=0.0341580(10,066.8)=344$ BOPD;

$q_o\ (550 \text{ psi})=0.0341580(1,440,000-302,500)^{0.650}$

$=295$ BOPD.

Using the dimensionless backpressure curve form in terms of $q_o/q_{o(\max)}$, $p_{wf}/\bar{p}_R$, and $n=0.650$,

$$q_o=70 \text{ BOPD; } \left(\frac{p_{wf}}{\bar{p}_R}\right)^2=\left(\frac{1,147}{1,200}\right)^2=0.9136;$$

$$\frac{q_o}{q_{o(\max)}}=\left[1-\left(\frac{p_{wf}}{\bar{p}_R}\right)^2\right]^{0.650}$$

$=(1-0.9136)^{0.650}=0.203579;$

$$q_{o(\max)}=\frac{70}{0.203579}=344 \text{ BOPD};$$

q_o at $p_{wf}=550$ psia.

$$\frac{q_o(550 \text{ psi})}{q_{o(\max)}}=\left[1-\left(\frac{550}{1,200}\right)^2\right]^{0.650}=0.857892;$$

$q_o=344(0.857892)=295$ BOPD.

Again, this example is based on field data where several rates were measured to establish the real IPR relationship of the well. The real absolute open flow of the well was 340 BOPD. This is 38% of the rate predicted by Vogel's IPR equation and 42% of the rate predicted by the backpressure equation with $n=1$. A value of $n=0.650$ as illustrated in this example is required to match the field data. A non-Darcy flow factor F_{Da} is indicated for this test.

Single-Phase and Two-Phase IPR Equation. Fetkovich[16] gives a general equation that treats flow both above and below the bubblepoint pressure for an undersaturated oil well.

$$q_o=J^*(\bar{p}_R-p_b)+J'(p_b{}^2-p_{wf}{}^2), \quad \ldots\ldots\ldots\ldots (78)$$

where

$$J'=J^*(\mu_o B_o)_{p_R,p_b}\left(\frac{a_2}{2}\right). \quad \ldots\ldots\ldots\ldots\ldots\ldots (79)$$

Assuming $(\mu_o B_o)$ is a constant value above the bubblepoint pressure equal to $(\mu_o B_o)_b$ (the basis of the constant PI assumption for flow above the bubblepoint pressure, p_b), then $a_2=1/[p_b(\mu_o B_o)_b]$ (see Appendix of Ref. 16).

Then

$$J'=\frac{J^*(\mu_o B_o)_b}{2p_b(\mu_o B_o)_b}=\frac{J^*}{2p_b}. \quad \text{(80)}$$

Substituting Eq. 80 into 78 we obtain the final form of the single-phase and two-phase IPR equation:

$$q_o=J^*(\bar{p}_R-p_b)+\frac{J^*}{2p_b}(p_b{}^2-p_{wf}{}^2). \quad \text{(81)}$$

Example Problem 7. The following example illustrates the method of computing inflow rates for flows both above and below the bubblepoint pressure of an undersaturated oil well.

An oil well is producing at a rate of 50 STB/D at a flowing BHP of 2,100 psia. The reservoir average shut-in pressure is 3,200 psia with a bubblepoint pressure of 1,800 psia.

Calculate the maximum possible flow rate, q_o, at $p_{wf}=0$ psig and the producing rate at 550 psia flowing BHP. (For flows above p_b, $J=J^*$.)

$$J=J^*=\frac{q_o}{(\bar{p}_R-p_{wf})},$$

therefore,

$$J^*=\frac{50}{(3{,}200-2{,}100)}=\frac{50}{1{,}100}$$

$$=0.045454 \text{ STB/D-psi}$$

and

$$q_o(15 \text{ psi})=J^*(\bar{p}_R-p_b)+\frac{J^*}{2p_b}(p_b{}^2-p_{wf}{}^2),$$

$$=0.045454(3{,}200-1{,}800)$$

$$+\frac{0.045454}{2(1{,}800)}(1{,}800^2-15^2),$$

$$=64+0.000012626(3{,}240{,}000-225),$$

$$=64+41=105.$$

This compares to 145 BOPD if the regular PI equation is assumed valid to 15 psia.

q_o at $p_{wf}=550$ psia

$$q_o(550 \text{ psi})=J^*(\bar{p}_R-p_b)+\frac{J^*}{2p_b}(p_b{}^2-p_{wf}{}^2)$$

$$=0.045454(3{,}200-1{,}800)$$

$$+\frac{0.045454}{2(1{,}800)}(1{,}800^2-550^2),$$

$$=64+0.000012626(3{,}240{,}000-302{,}500),$$

$$=64+37=101 \text{ BOPD}.$$

The additional 535-psi pressure drop from 550 psia to 15 psia results in only 4 BOPD increase. It is significant to point out that if several flows, all with flowing pressure p_{wf} below the bubblepoint pressure p_b, were calculated using the above equation and example and then plotted as a backpressure curve but with $\bar{p}_R{}^2-p_{wf}{}^2$, it would indicate a value of $n=0.820$. We would have an indicated n less than 1.0 without a non-Darcy flow term F_{Da}. With the uncertainty involved in really knowing the *true* bubblepoint pressure of a particular well, we could obtain test n values less than 1.0 without non-Darcy flow existing.

To illustrate more clearly a case of drawdown data obtained at flowing pressures below the bubblepoint pressure to obtain J^*, we will use the 550 psia rate obtained above and the previously specified data. Actual unrounded calculated rate is 100.73 BOPD.

$$q_o=J^*\left[(\bar{p}_R-p_b)+\frac{(p_b{}^2-p_{wf}{}^2)}{2p_b}\right]$$

$$J^*=\frac{q_o}{\left[(\bar{p}_R-p_b)+\frac{(p_b{}^2-p_{wf}{}^2)}{2p_b}\right]},$$

$$=\frac{100.73}{\left[(3{,}200-1{,}800)+\frac{(3{,}240{,}000-302{,}500)}{2(1{,}800)}\right]},$$

$$=\frac{100.73}{(1{,}400+816)}=\frac{100.73}{2{,}216}$$

$$=0.045450 \text{ STB/D-psi (good check)}.$$

Future Inflow Performance. Standing[17] presented a method for adjusting IPR by using Vogel's equation from a measured condition to a future reservoir pressure $\bar{p}_R$. It is based on the fact that PI can be defined uniquely only at a zero drawdown, $p_{wf}\rightarrow\bar{p}_R$.

$$J^*=\lim_{\Delta p\rightarrow 0} J. \quad \text{(82)}$$

Applying the limit condition using Vogel's equation yielded

$$J^*=\frac{1.8q_{o(\max)}}{\bar{p}_R}. \quad (83)$$

Using the same approach with the backpressure equation and $n=1$,

$$\frac{q_o}{q_{o(\max)}}=\left[1-\left(\frac{p_{wf}}{\bar{p}_R}\right)^2\right]^{1.0}$$

which yields

$$J^*=\frac{2q_{o(\max)}}{\bar{p}_R}. \quad (84)$$

If we define $q_{o^*(\max)}$ as that absolute open flow potential we would obtain, assuming conventional Δp PI were used,

$$q_{o^*(\max)}=J^*(\bar{p}_R-0)$$

and

$$q_{o^*(\max)}=J^*\bar{p}_R=2q_{o(\max)}. \quad (85)$$

Note that the "real" $q_{o(\max)}$ is ½ that assuming a Δp productivity index relationship. This is more clearly seen from Fig. 34.5 and Eq. 86. In terms of the Evinger-Muskat equation,

$$q_o=J^*\int_{p_{wf}}^{\bar{p}_R}\frac{k_{ro}}{(\mu_oB_o)}dp=J^*A_c, \quad (86)$$

where A_c=area under curve.

For the $n=1.0$ IPR relationship, the area under the curve (A, C, D) is exactly ½ that area (A, B, C, D) assuming Δp PI relationship when $p_{wf}=0$.

Example Problem 8. Using Standing's example data we will (1) calculate present J^*_p from present flow data, (2) adjust J^*_p to a future J^*_f, and (3) calculate a future rate at $p_{wf}=1{,}200$ psig.

The following was given in Standing's example.[17] The present PI, J, was determined to be 0.92 at a flow rate of 400 BOPD with $p_{wf}=1{,}815$ psig. Average reservoir pressure, $\bar{p}_R$, at this time is 2,250 psig. Future reservoir pressure $\bar{p}_R$ will be 1,800 psig. $k_{ro}/(\mu_oB_o)=0.2234$ present and 0.1659 future.

$$q_{o(\max)}=\frac{q_o}{\left[1-\left(\frac{p_{wf}}{\bar{p}_R}\right)^2\right]}$$

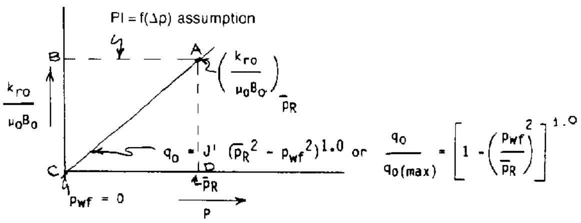

Fig. 34.5—Simple pressure function for Δp_2 relationship and $n=1$.

$$=\frac{400}{\left[1-\left(\frac{1{,}830}{2{,}265}\right)^2\right]}=1{,}152 \text{ BOPD},$$

$$J^*=\frac{2q_{o(\max)}}{\bar{p}_R}=\frac{2(1{,}152)}{2{,}265}=1.017,$$

$$J^*_f=J^*_p\frac{\left(\frac{k_{ro}}{\mu_oB_o}\right)_f}{\left(\frac{k_{ro}}{\mu_oB_o}\right)_p}=1.017\frac{0.1659}{0.2234}=0.755,$$

$$q_{o(\max)f}=\frac{J^*_f(\bar{p}_R)}{2}=\frac{0.755(1{,}800+15)}{2}=685 \text{ BOPD},$$

and

$$q_{of}(1{,}200 \text{ psig})=q_{o(\max)f}\left[1-\left(\frac{p_{wf}}{\bar{p}_R}\right)^2\right]$$

$$=685\left[1-\left(\frac{1{,}215}{1{,}815}\right)^2\right]=378 \text{ BOPD}.$$

Multiphase Flow

Introduction

Much has been published in the literature on the vertical simultaneous flow of two or more fluids through a pipe. The general problem of predicting the pressure drop for the simultaneous flow of gas and liquid is complex. The problem consists of being able to predict the variation of pressure with elevation along the length of the flow string for known conditions of flow. The ability to do this in the case of flowing oil wells provides a means of evaluating the effects of tubing size, flow rate, BHP, and a host of other variables on one another. In the case of gas lift installations in oil wells, it would be particularly useful in designing the installation and providing such information as the optimum depth, pressure, and the rate at which to inject the gas, the horsepower requirements to lift the oil, and the effect of production rate and tubing size on these quantities. In other words, a means of systematically studying the effects of the different variables upon one another.

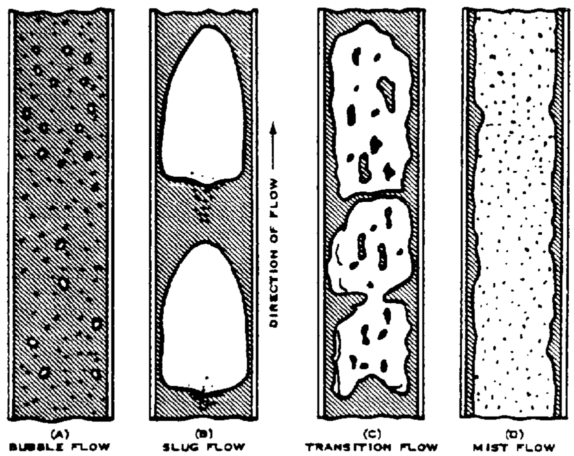

Fig. 34.6—Flow regime classifications for vertical two-phase flow.

Multiphase flow may be categorized into four different flow configurations or flow regimes, consisting of bubble flow, slug flow, slug-mist transition flow, and mist flow. In bubble flow, the liquid is continuous with the gas phase existing as bubbles randomly distributed (Fig. 34.6). The gas phase in bubble flow is small and contributes little to the pressure gradient except by its effect on the density. A typical example of bubble flow is the liberation of solution gas from an undersaturated oil at and above the point in the flow string where its bubblepoint pressure is reached.

In slug flow, both the gas and liquid phases significantly contribute to the pressure gradient. The gas phase in slug flow exists as large bubbles almost filling the pipe and separated by slugs of liquid. The gas bubbles are rounded on their leading edge, fairly flat on their trailing edge, and are surrounded on their sides by a thin liquid film. Liquid entrainment in the gas phase occurs at high flow velocities and small gas bubbles occur in the liquid slug. The velocity of the gas bubbles is greater than that of the liquid slugs, thereby resulting in a liquid holdup that not only affects well friction losses but also flowing density. Liquid holdup is defined as the in-situ flowing volume fraction of liquid. Slug flow accounts for a large percentage of two-phase production wells and, as a result, a good deal of research has been concentrated on this flow regime.

In transition flow, the liquid slugs between the gas bubbles essentially disappear, and at some point the liquid phase becomes discontinuous and the gas phase becomes continuous. The pressure losses in transition flow are partly a result of the liquid phase, but are more the result of the gas phase.

Mist flow is characterized by a continuous gas phase with liquid occurring as entrained droplets in the gas stream and as a liquid film wetting the pipe wall. A typical example of mist flow is the flow of gas and condensate in a gas condensate well.

Complete sets of pressure traverses for specific flow conditions and oil and gas properties have been published by service companies and others. These pressure gradient curves can be used for quick hand calculations.

Theoretical Considerations

As discussed in the Theoretical Basis section, the basis of any fluid-flow calculation consists of an energy balance on the fluid flowing between any two points in the system under consideration. The energy entering the system by virtue of the flowing fluid must equal the energy leaving the system plus the energy interchanged between the fluid and its surroundings.

The pressure drop in a vertical pipe associated with either single- or multiphase flow is given by

$$-\mathrm{d}p=\frac{\tau_f \mathrm{d}D}{144}+\frac{g\rho}{144 g_c}\mathrm{d}D+\frac{g v}{144 g_c}\mathrm{d}v, \quad \ldots\ldots (87)$$

where

p = pressure, psia,
τ_f = friction loss gradient, lbf/sq ft-ft,
D = depth, ft,
g = acceleration of gravity, ft/sec^2,
g_c = gravitational constant, (ft-lbm)/(lbf sec^2),
ρ = fluid density, lbm/cu ft, and
v = fluid velocity, ft/sec.

Eq. 87 states that the fluid pressure drop in a pipe is the combined result of friction, potential energy, and kinetic energy losses.

The friction loss gradient and average density term for multiphase flow are evaluated using specific relationships for each flow regime. The kinetic energy term is usually small except for large flow rates. Duns and Ros[18] have shown that for two-phase flow the kinetic energy term is significant only in the mist flow regime. Under this flow condition, $v_g \gg v_L$, and the kinetic energy term can be expressed as

$$\frac{\rho v}{g_c}\mathrm{d}v=-\frac{w_t q_g}{g_c A^2}\frac{\mathrm{d}p}{p}, \quad \ldots\ldots (88)$$

where

A = pipe area, sq ft,
w_t = total mass flow rate, lbm/sec, and
q_g = gas volumetric flow rate, cu ft/sec.

Eq. 87 now can be written in difference form for any depth increment, i, by assuming an average temperature and pressure exists over the increment. Making this assumption we have

$$\Delta p_i=\frac{1}{144}\left(\frac{\bar{\rho}+\tau_f}{1-\dfrac{w_t q_g}{4637 A^2 \bar{p}}}\right)\Delta D_i, \quad \ldots\ldots (89)$$

where

$\bar{\rho}$ = average fluid density, lbm/cu ft,
Δp_i = pressure drop for increment i, psi,
$\bar{p}$ = average pressure, psia, and
ΔD_i = the ith depth increment, ft.

Eq. 89 can be solved incrementally either by setting Δp_i and solving for ΔD_i or by setting ΔD_i and solving for Δp_i. Since pressure usually has more effect on average fluid properties than temperature and since temperature can be expressed as a function of depth, Δp_i should be set and ΔD_i calculated. The calculation procedure described here is an iterative process for each section and generally is programmed for solution on a computer.

Correlations

Since the original work in this area, which was presented by Poettmann and Carpenter,[33] several studies have been undertaken to collect additional experimental multiphase flow data and to develop new multiphase pressure drop correlations.[18-29] Also, various statistical studies have been performed comparing recent multiphase flow correlations[30-32] for large sets of flowing and gas lift cases.

Espanol *et al.*[30] selected the Hagedorn and Brown,[24] Duns and Ros,[18] and Orkiszewski[25] methods as three of the best correlations for calculating multiphase pressure drops. An analysis of results calculated on 44 wells was used to determine the best overall correlation. This work concluded that the Orkiszewski correlation was the most accurate method over a large range of well conditions and it was the only correlation of the three considered suitable for evaluating three-phase flow for wells producing significant quantities of water.

Lawson and Brill[32] point out that the Poettmann and Carpenter method is still a base line for comparing new multiphase flow correlations. Their original work is based on flow conditions similar to those found in many gas lift conditions and, therefore, is briefly discussed.

Poettmann and Carpenter.[33] Poettmann and Carpenter used data on flowing and gas lift wells to correlate the combined energy losses resulting from liquid holdup, frictional effects caused by the surface of the tubing, and other energy losses as a function of flow variables.

No attempt was made to evaluate the various components making up the total energy loss. The flowing fluid was treated as a single homogeneous mass, and the energy loss was correlated on this basis. A total flowing density or specific volume was used rather than an in-situ density or specific volume. That is, the energy of the fluid entering and leaving the tubing is a function of the pressure-volume properties of the total fluid entering and leaving the tubing, and not of the pressure-volume properties of the fluid in place, which would be different because of slippage or liquid-holdup effects. Lastly, in calculating flowing density or flowing specific volume, mass transfer between phases as the fluid flows up the tubing was taken into consideration, as well as the entire mass of the gas and liquid phases.

Viscosity as a correlating function was neglected. The degree of turbulence is of such a magnitude, in general, for a two-phase flowing oil well that the portion of the total energy loss resulting from viscous shear is negligible. This is not surprising since it is also true for single-phase turbulent flow. There the energy loss is independent of the physical properties of the flowing fluid. A number of others* working on the same problem of multiphase flow have made the same observation.

Baxendell extended Poettmann and Carpenter's correlation by using large-volume flow data from wells on casing flow.[34] A detailed discussion of the Poettmann and Carpenter development can be found in the original 1962 edition of this handbook and in Ref. 33. The Poettmann and Carpenter correlation has served as the take-off point for many of the newer multiphase flow correlations.

Orkiszewski. To obtain a set of calculation procedures covering all flow regimes in two-phase flow, Orkiszewski[25] made a thorough review of the literature, tested various methods against a few sets of experimental data by hand calculations, and then selected the two methods, Griffith and Wallis[19] and Duns and Ros,[18] for his final evaluation. Orkiszewski programmed both methods and tested them against data from 148 wells. Neither method was accurate over the entire set of flow conditions. Griffith and Wallis's method, however, appeared to provide the better foundation for a general solution in slug flow, and, thus, Orkiszewski elected to modify their work.

Orkiszewski called his calculation procedures the Modified Griffith and Wallis method since their work was involved strictly with fully developed slug flow and since 95% of the 148 wells used by Orkiszewski in developing his method were in slug flow. Duns and Ros' method was used for mist flow and partly for transition flow since it appeared to be more fundamental than the Lockhart and Martinelli[35] method recommended by Griffith.

Orkiszewski's method essentially establishes which flow regime is present and then applies (1) Griffith's procedure for bubble flow, (2) Griffith's procedure modified by a liquid distribution coefficient parameter based on field data for slug flow, (3) a combination of the modified Griffith method and the Duns and Ros method for transition flow, or (4) Duns and Ros' method for mist flow. Accuracy claimed for this correlation is about ±10% for a wide range of flow conditions.

The determination of which flow regime applies for a given pipe segment is accomplished by checking the various dimensionless groups that define the boundaries of each flow regime (Fig. 34.7). Griffith and Wallis are responsible for defining the boundary between the bubble and slug flow regimes. Duns and Ros have defined the boundaries between the slug and transition flow regimes and between the transition and mist flow regimes. These boundaries are given by the inequalities listed below.

1. For the bubble flow regime, the boundary limits are $q_g/q_t < L_B$.
2. For the slug flow regime, the boundary limits are $q_g/q_t > L_B$, $v_{gD} < L_S$.
3. For the transition flow regime, the boundary limits are $L_M > v_{gD} > L_S$.
4. For the mist flow regime, the boundary limits are $v_{gD} > L_M$.

In these equations the subscripts B, M, and S indicate bubble, mist, and slug flow, respectively.

*Early investigators of this problem were T.V. Moore and H.D. Wilde Jr., "Experimental Measurement of Slippage in Flow Through Vertical Pipes," *Trans.*, AIME (1931) **92**, 296–313; and T.V. Moore and R.J. Schilthuis, "Calculation of Pressure Drops in Flowing Wells," *Trans.*, AIME (1933) **103**, 170–86.

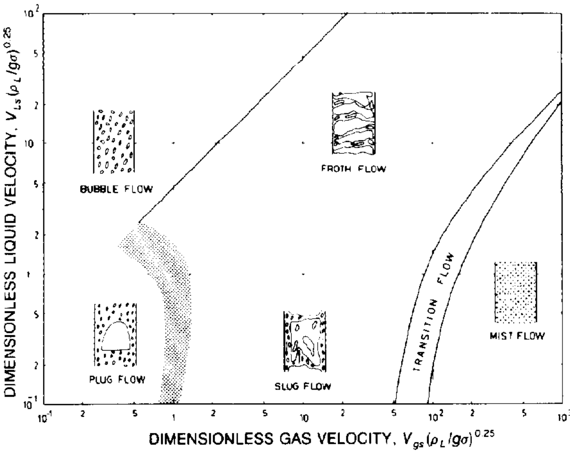

Fig. 34.7—Flow regime map.

These dimensionless groups are given by the following set of equations.

$$v_{gD}=\frac{q_g[\rho_L/(g\sigma)]^{0.25}}{A}, \quad \text{(90)}$$

at the bubble-slug boundary

$$L_B=1.071-\frac{0.2218v_t^2}{d_H}, \quad \text{(91)}$$

but

$$L_B \geq 0.13,$$

at the slug-transition boundary

$$L_S=50+\frac{36v_{gD}q_L}{q_g}, \quad \text{(92)}$$

and at the transition-mist boundary

$$L_M=75+84\left(\frac{v_{gD}q_L}{q_g}\right)^{0.75}, \quad \text{(93)}$$

where

v_{gD} = dimensionless gas velocity,
v_t = total fluid velocity (q_t/A), ft/sec,
ρ_L = liquid density, lbm/cu ft,
σ = liquid surface tension, lbm/sec^2,
L = flow regime boundary, dimensionless,
d_H = hydraulic pipe diameter, ft,
q_g = gas flow rate, cu ft/sec,
g = acceleration of gravity, ft/sec^2, and
A = flow area of pipe, sq ft.

The average density and friction loss gradient is defined later for each of the four possible flow regimes. These terms are evaluated for each pipe segment and are then substituted into Eq. 89 to calculate the pressure drop over the segment.

Bubble Flow. The average flowing density in *bubble flow* is calculated from the following equation, which volumetrically weights the gas and liquid densities.

$$\bar{\rho}=\rho_g f_g+(1-f_g)\rho_L. \quad \text{(94)}$$

The flowing gas fraction, f_g, in bubble flow is given by

$$f_g=\frac{1}{2}\left[1+\frac{q_t}{v_sA}-\sqrt{\left(\frac{1+q_t}{v_sA}\right)^2-\frac{4q_g}{v_sA}}\right], \quad \text{(95)}$$

where the slip velocity, v_s, is the difference between the average gas and liquid velocities. Griffith suggests the use of an approximate value of $v_s=0.8$ ft/sec for bubble flow.

The friction loss gradient for bubble flow is based on single-phase liquid flow.

$$\tau_f=\frac{f\rho_L v_L^2}{2g_c d_H \cos\theta}, \quad \text{(96)}$$

where

$$v_L=\frac{q_L}{A(1-f_g)}. \quad \text{(97)}$$

The friction factor, f, in Eq. 96 is the standard Moody[2] friction factor, which is a function of Reynolds number and relative roughness factor. The Reynolds number that is used for bubble flow is the liquid Reynolds number.

$$N_{Re}=\frac{1488\rho_L d_H v_L}{\mu_L}, \quad \text{(98)}$$

where d_H is the hydraulic pipe diameter (4A/wetted perimeter), ft, and μ_L is the liquid viscosity, cp.

Slug Flow. The average density term for *slug flow* is expressed as

$$\bar{\rho}=\frac{w_t+\rho_L v_b A}{q_t+v_b A}+\delta\rho_L. \quad \text{(99)}$$

Eq. 99, with the exception of its last term, is equivalent to the average density term derived by Griffith and Wallis. The last term of Eq. 99 was added by Orkiszewski and contains a parameter, δ, that was correlated from oilfield data. The slip or bubble rise velocity, v_b, for slug flow was correlated by Griffith and Wallis and is given by

$$v_b=C_1C_2\sqrt{gd_H}. \quad \text{(100)}$$

The coefficient C_1 is the bubble-rise coefficient for bubbles rising in a static column of liquid. Values of C_1 have been determined theoretically by Dumitrescu[36] and experimentally by Griffith and Wallis[19] as a function of bubble Reynolds number, Fig. 34.8, where

$$N_{Re_b}=\frac{1488\rho_L d_H v_b}{\mu_L}. \quad \text{(101)}$$

The coefficient C_2 is a function of liquid velocity and, when multiplied by C_1, represents the bubble-rise coefficient for bubbles rising in a flowing liquid. The coefficient C_2 has been determined experimentally by Griffith and Wallis[19] and is correlated as a function of both bubble Reynolds number, N_{Re_b}, and liquid Reynolds number (Fig. 34.9), where

$$N_{Re} = \frac{1488 \rho_L d_H v_t}{\mu_L}. \quad \ldots\ldots (102)$$

When Reynolds numbers larger than 6,000 are encountered, v_b can be evaluated from the following equations, which were developed by Orkiszewski and based on the work of Nicklin *et al.*[20] For bubble Reynolds numbers, N_{Re_b}, less than 3,000,

$$v_b = [0.546 + 8.74(10^{-6}) N_{Re}] \sqrt{g d_H}. \quad \ldots\ldots (103)$$

When bubble Reynolds number is between 3,000 and 8,000,

$$v_b = 0.5 v_{bi} + 0.5 \left(v_{bi}^2 + \frac{13.59 \mu_L}{\rho_L \sqrt{d_H}} \right)^{0.5}, \quad \ldots\ldots (104)$$

where

$$v_{bi} = [0.251 + 8.74(10^{-6}) N_{Re}] \sqrt{g d_H}. \quad \ldots\ldots (105)$$

For bubble Reynolds numbers greater than 8,000,

$$v_b = [0.35 + 8.74(10^{-6}) N_{Re}] \sqrt{g d_H}. \quad \ldots\ldots (106)$$

The friction loss gradient term for slug flow is the result of Orkiszewski's work and is given by

$$\tau_f = \frac{f \rho_L v_t^2}{2 g_c d_H \cos\theta} \left(\frac{q_L + v_b A}{q_t + v_b A} + \delta \right). \quad \ldots\ldots (107)$$

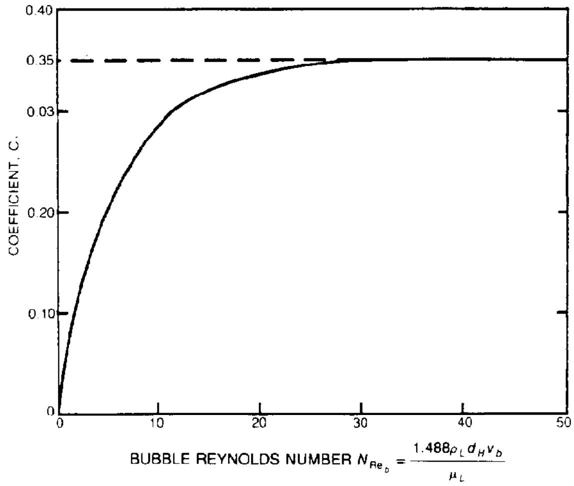

Fig. 34.8—Bubble-rise coefficient for bubbles rising in a static liquid column vs. bubble Reynolds number.

The friction factor in Eq. 107 is a function of relative roughness and the Reynolds number given by Eq. 102.

Orkiszewski defined the parameter δ, which appears in Eqs. 99 and 107 as a liquid distribution coefficient. This coefficient implicitly accounts for the following physical phenomena.

1. Liquid is distributed not only in the slug and as a film around the gas bubble but also as entrained droplets inside the gas bubble.
2. The friction loss has essentially two contributions, one from the liquid slug and the other from the liquid film.
3. The bubble rise velocity approaches zero as mist flow is approached.

Liquid distribution coefficient, δ, was correlated as a function of liquid viscosity, hydraulic radius, and total velocity and may be evaluated by one of the following empirical equations.

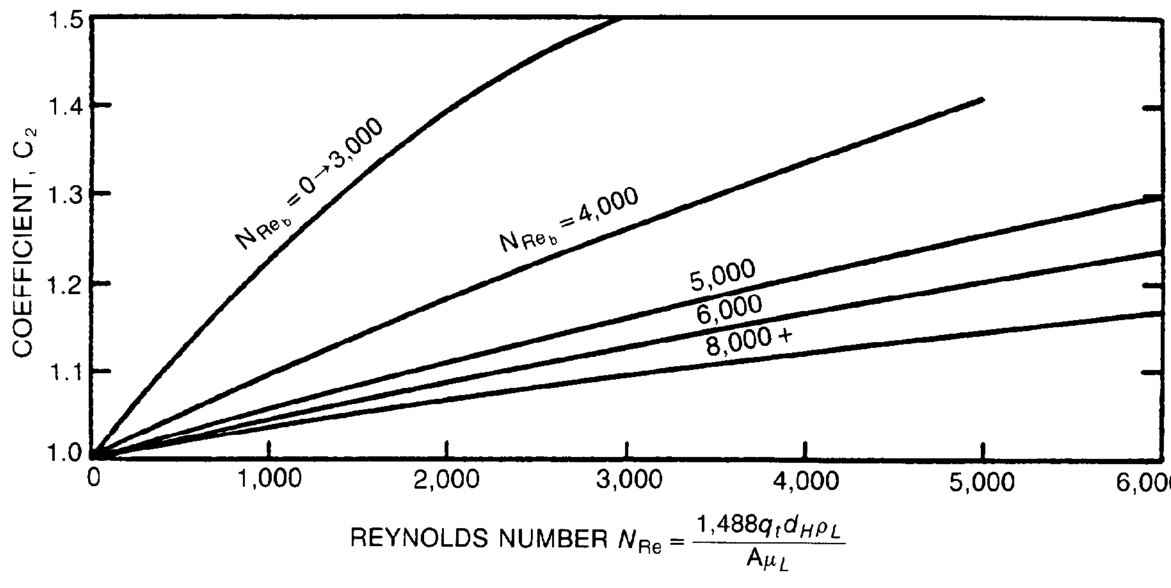

Fig. 34.9—Bubble-rise coefficient accounting for bubbles rising in a flowing liquid vs. Reynolds number.

Continuous Oil Phase. When $v_t<10$,

$$\delta=\frac{0.0127}{d_H{}^{1.415}}\log(\mu_L+1)-0.284+0.167\log v_t$$

$$+0.113\log d_H. \qquad (108)$$

When $v_t>10$,

$$\delta=\frac{0.0274}{d_H{}^{1.371}}\log(\mu_L+1)+0.161+0.569\log d_H$$

$$-\log v_t\left[\frac{0.01}{d_H{}^{1.571}}\log(\mu_L+1)\right.$$

$$\left.+0.397+0.63\log d_H\right]. \qquad (109)$$

Continuous Water Phase. When $v_t<10$,

$$\delta=\frac{0.013}{d_H{}^{1.38}}\log\mu_L-0.681$$

$$+0.232\log v_t-0.428\log d_H. \qquad (110)$$

When $v_t>10$,

$$\delta=\frac{0.045}{d_H{}^{0.799}}\log\mu_L-0.709-0.162\log v_t$$

$$-0.888\log d_H. \qquad (111)$$

Eqs. 108 through 111 are constrained by the following limits, which eliminate pressure discontinuities between flow regimes. When $v_t<10$, $\delta\geq-0.065v_t$, and when $v_t>10$,

$$\delta\geq-\frac{v_bA(1-\rho/\rho_t)}{q_t+v_bA}.$$

Transition Flow. The Duns and Ros method for calculating average flowing density and friction loss gradient in *transition flow* is used. They evaluated ρ and τ_f by linearly weighting the values obtained from slug and mist flow with dimensionless gas velocity, v_{gD}, and the dimensionless boundaries defining transition flow, L_M and L_S. The average density term is defined as

$$\bar{\rho}=\left(\frac{L_M-v_{gD}}{L_M-L_S}\right)\bar{\rho}_S+\left(\frac{v_{gD}-L_S}{L_M-L_S}\right)\bar{\rho}_M, \qquad (112)$$

where subscripts M and S are mist and slug flow conditions, respectively. Similarly, the friction loss gradient is defined as

$$\tau_f=\left(\frac{L_M-v_{gD}}{L_M-L_S}\right)\tau_{fS}+\left(\frac{v_{gD}-L_S}{L_M-L_S}\right)\tau_{fM}. \qquad (113)$$

Mist Flow. In *mist flow* the slip between the gas and liquid phases is essentially zero. The fraction of gas flowing can be expressed, therefore, as

$$f_g=\frac{q_g}{q_g+q_L}. \qquad (114)$$

Average flowing density is given by

$$\bar{\rho}=(1-f_g)\rho_L+f_g\rho_g. \qquad (115)$$

The friction loss gradient for mist flow is primarily a result of the gas phase and is given by

$$\tau_f=\frac{f\rho_g v_{gs}{}^2}{2g_c d_H\cos\theta}, \qquad (116)$$

where v_{gs} is the superficial gas velocity and f is a function of the gas Reynolds number,

$$N_{Re}=1488\frac{\rho_g d_H v_{gs}}{\mu_g}, \qquad (117)$$

and a modified relative roughness factor, ϵ/d_H, which was developed by Duns and Ros. The roughness factor for mist flow is a function of the liquid film wetting the pipe walls and is given by the following set of equations and constraints. Let

$$N=4.52(10^{-7})(v_{gs}\mu_L/\sigma)^2(\rho_g/\rho_L), \qquad (118)$$

where N is a dimensionless number. Then for $N<0.005$,

$$\frac{\epsilon}{d_H}=\frac{34\sigma}{\rho_g v_{gs}{}^2 d_H} \qquad (119)$$

and for $N>0.005$,

$$\frac{\epsilon}{d_H}=\frac{174.8\sigma(N)^{0.302}}{\rho_g v_{gs}{}^2 d_H}. \qquad (120)$$

Eqs. 119 and 120 are limited by upper and lower bounds for ϵ/d_H of 0.001 and 0.05.

Camacho[31] studied 111 wells with high gas/liquid ratios and concluded that Orkiszewski's method performed better when mist flow calculations were used for gas/liquid ratios greater than 10,000. Obviously, if this approach is taken, an appropriate transition zone between slug and mist flow should be used to avoid abrupt pressure gradient changes. In another study, Gould *et al.*[27] also indicate that the onset of mist flow should occur at lower dimensionless gas velocities, especially for dimensionless liquid velocities less than 0.1.

Continuous-Flow Gas Lift Design Procedures

Gas lift[28,33,37] is a method of artificial lift that uses the compressional energy of a gas to lift the reservoir fluid (see Chap. 5). The prime requisite is an adequate source of gas at a desired pressure and volume.

Wells having high water/oil ratios (WOR) and high productivity indices (that is, producing large volumes of fluid with high sustaining reservoir pressures) can be efficiently gas lifted through the tubing or the well annulus. Quite often it is necessary to produce very large volumes of water to obtain economic rates of oil production. Situations are known where it is possible to gas lift economically as much as 5,000 to 10,000 B/D total fluid, with the oil present being 1% of the total fluid produced and the rest being water. In applying the correlations to gas lift design calculations, the following procedure is recommended.

1. Establish the flow characteristics of the well—i.e., productivity index, WOR, gas/oil ratio (GOR), fluid properties, tubing size, etc.
2. Calculate the pressure traverses below the injection point for the range of flow rates.
3. Calculate the pressure traverses above the point of injection for different injection GOR's, holding the surface tubing or casing pressure constant.

From these three steps, as illustrated in Fig. 34.10, the horsepower requirements, pressure at injection point, depth of injection, and injection GOR's for a given rate of production, tubing size, and tubing or casing pressure can be calculated.

For a given set of well conditions and fluid production, there is an optimum depth and injection pressure that result in minimum horsepower requirements. In some cases, the optimal injection depth will be at the total depth of the well. There are two ranges of operation in gas lifting a reservoir fluid. One is an inefficient range characterized by high GOR and high horsepower requirements, and the other is an efficient range characterized by low GOR and low horsepower requirements. A plot of GOR vs. injection pressure is shown in Fig. 34.11.

In the inefficient range of operation, gas literally is "blown" through the flow string. The efficient range is to the left of the minimum injection pressure, and the inefficient range to the right. Inefficient and efficient ranges of operation have been observed in the laboratory on experimental gas lift involving short lengths of tubing.[38-40] One investigator used a large amount of field data from a California field to develop empirically curves similar to those shown in Fig. 34.11 but had no way of predicting these curves for other fields where the physical properties of the fluids and the production data were different.[41] In a plot of horsepower requirement vs. injection pressure (Fig. 34.12) the horsepower generally passes through a minimum value, which represents the maximum efficiency of the operation. Another interesting result of these gas lift calculations has been to show that the lower the surface pressure of the flow string that can be maintained consistent with efficient surface operations, the less will be the horsepower required to lift the reservoir fluid.

The use of the calculation procedure can best be expressed by use of a typical example problem.[42]

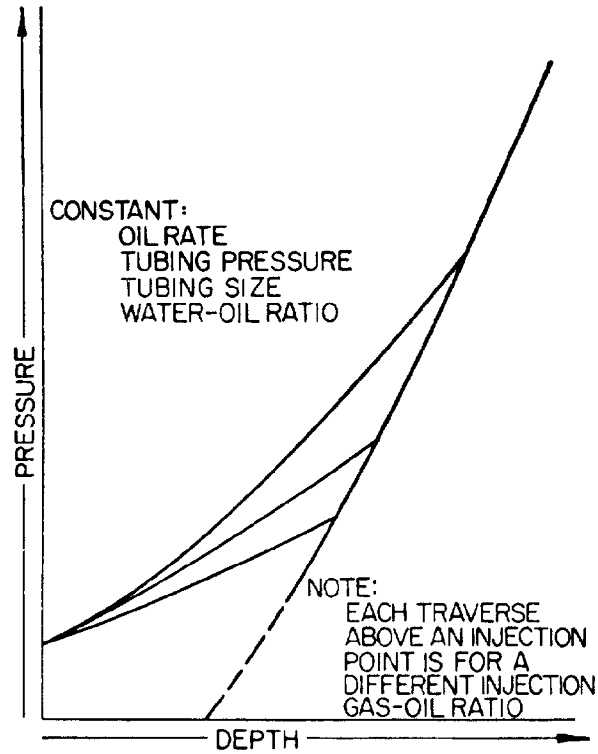

Fig. 34.10—Pressure traverse in gas-lift well.

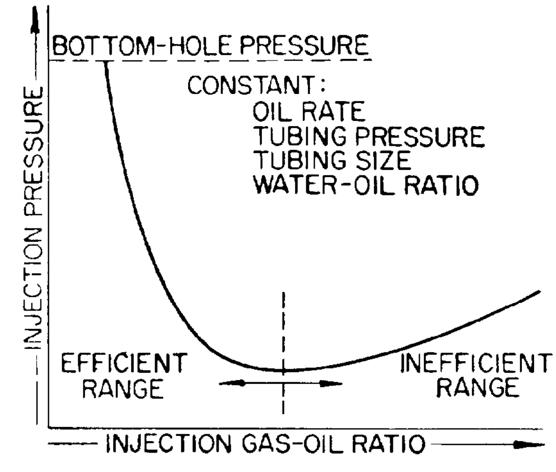

Fig. 34.11—Effect of injection pressure on injection GOR.

Example Problem 9. It is desired to gas lift a well by flowing through the annulus. The well has a productivity index of 10.0 bbl total liquid per day per psi pressure drop. The static reservoir pressure is 3,800 psia at a well depth of 10,000 ft. The WOR is 18.33. Other pertinent information is as follows.

Tubing ID (2½ in. nominal, 6.5 lbm/ft)=2.441 in.; tubing OD (2½ in. nominal, 6.5 lbm/ft)=2.875 in.; casing ID (7 in. nominal, 26 lbm/ft)=6.276 in.; casing pressure=100 psia; average flowing temperature in annulus above injection depth=155°F; average flowing temperature in annulus below injection depth=185°F; average flowing temperature in tubing=140°F; gravity of stock-tank oil at 60°F=0.8390; gravity of separator gas (air=1.0)=0.625; gravity of produced water=1.15; $B=0.0000723p+1.114$; $R_s=0.1875p+17$; and $R=600$ cu ft/bbl oil.

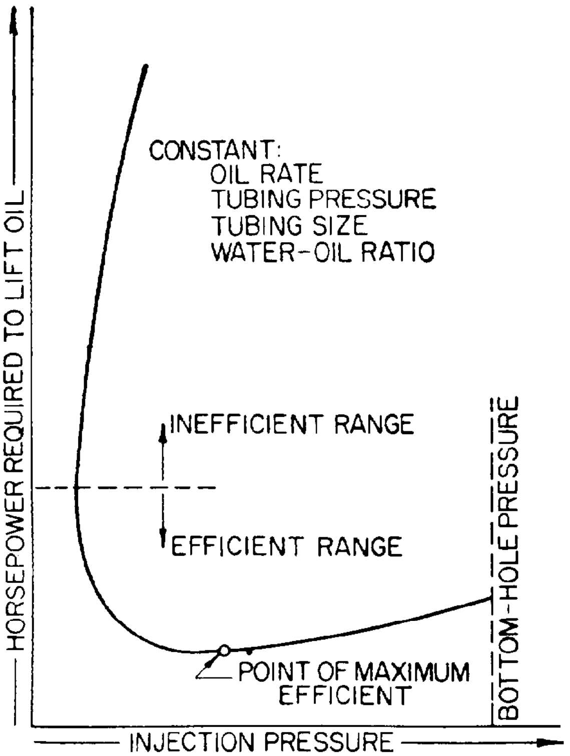

Fig. 34.12—Effect of injection pressure on horsepower requirements.

Calculate the variation of injection GOR with injection pressure and injection depth for a total liquid production rate of 4,000 B/D. Calculate the horsepower requirements to lift the oil as a function of injection pressure.

The solution of the problem involves the following steps.

1. Calculate the pressure traverse below the point of gas injection.
2. Calculate the pressure traverses above the point of gas injection for various GOR's.
3. Solve 1 and 2 simultaneously to determine the depth of injection for various injection GOR's and a casing pressure of 100 psia.
4. Calculate the theoretical adiabatic horsepower required to compress the gas from 100 psia to the injection-point pressure.

The first step in the solution of this problem is the calculation of the flowing density of the three-phase fluid produced into the well as a function of the pressure. Using Fig. 34.13, the differential pressure gradients were determined as a function of fluid density and, therefore, pressure. These calculations are illustrated in Table 34.6. These results then were placed on a plot of d*D*/d*p* vs. *p*. The depth traveled by the fluid flowing from the BHP to any lower pressure was determined by integrating this curve. In this way, Curve A in Fig. 34.14 was determined.

The second step of the solution was carried out mechanically the same as the first step, with the exceptions that the fluid densities were calculated for injection GOR's of 3,000, 3,500, 4,000, 5,000, and 7,500 scf/bbl, and that the integrations were carried out from the wellhead casing pressure of 100 psia to the pressures farther down the casing. The results of these calculations are shown in Fig.

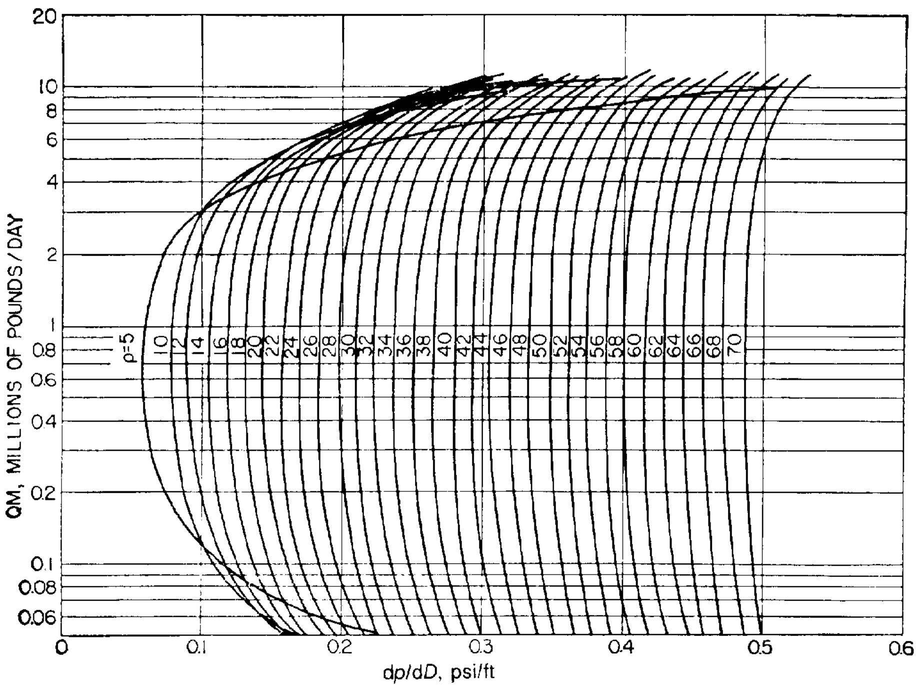

Fig. 34.13—Calculation of pressure traverses for flow in annulus. Tubing size is 2½ in. nominal (6.5 lbm/ft, 2.441-in. ID, 2.875-in. OD). Casing size is 7.0 in. nominal (26 lbm/ft, 6.276-in. ID).

TABLE 34.6—CALCULATION OF THE PRESSURE TRAVERSE BELOW THE POINT OF GAS INJECTION

$$q_o = \frac{4.000}{19.33} = 206.9 \text{ B/D}$$

$$q_o m = 1.594 \times 10^6 \text{ lbm/D}$$

$$\rho = \frac{m}{V_m} = \frac{7701.5}{5.61B + \dfrac{18.29(600 - R_s)}{p/z} + 102.8} \text{ lbm/cu ft}$$

Flowing BHP = 3,400 psia

Establishing p vs. $1/dp/dD$

p	B	R_s	p/z	ρ	dp/dD	$1/dp/dD$
3,400	1.339	—	3,800	69.8	0.487	2.053
3,000	1.331	588	3,440	69.8	0.487	2.053
2,000	1.259	392	2,270	69.0	0.481	2.079
1,000	1.286	205	1,078	66.3	0.460	2.174
500	1.150	110.8	520.8	60.9	0.425	2.353

$$\int_{3,400}^{p_2} \frac{dD}{dp} dp$$

p	$Dp_1 - Dp_2$	ΔD	D (ft)
3,400	—	0	10,000
3,000	− 821.2	− 821.2	9,179
2,500	−1,028.5	−1,849.7	8,150
2,000	−1,035.0	−2,884.7	7,115
1,500	−1,048.5	−3,933.2	6,066
1,000	−1,071.5	−5,004.7	4,995
500	−1,121.0	−6,125.7	3,874

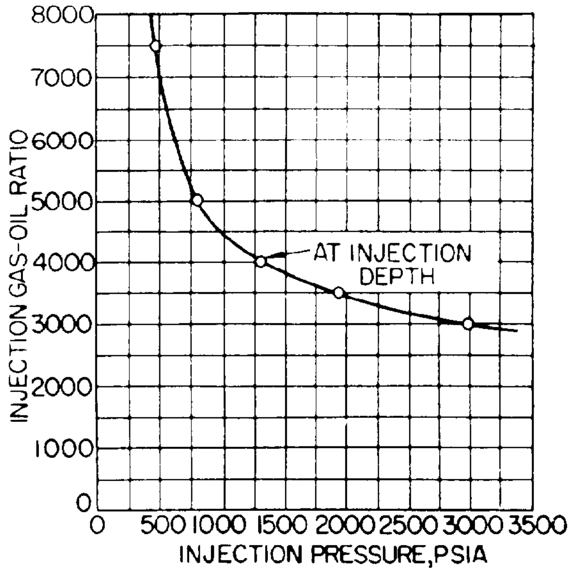

Fig. 34.15—Injection GOR vs. injection pressure.

34.14 as curves B, C, D, E, and F. The intersections of these curves with Curve A represent the injection points for these flow rates and injection GOR's.

The injection GOR is plotted as a function of the injection pressure at injection depth in Fig. 34.15. For the conditions of this example problem, it will be noted that the injection pressure continually decreases as the GOR is increased from 3,000 to 7,500 scf/bbl. Fig. 34.16 shows the relationship between injection depth and injection GOR. This plot shows that, as the injection GOR is decreased, the point of injection is moved down the hole.

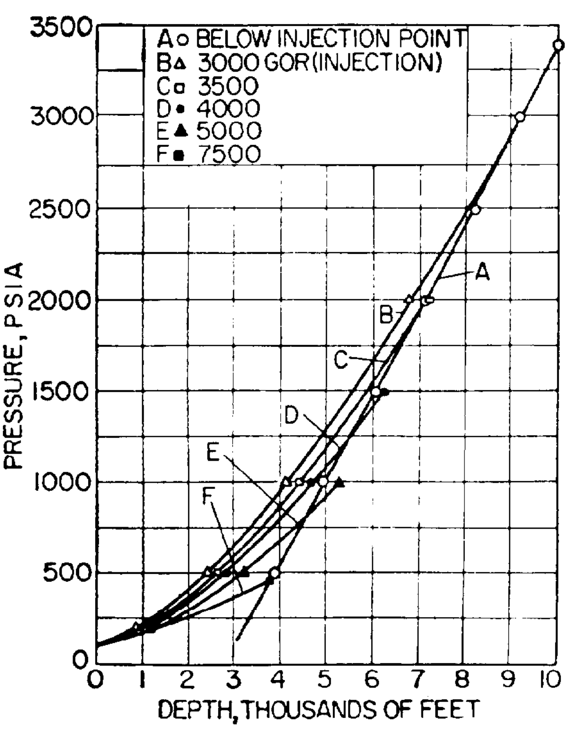

Fig. 34.14—Pressure vs. depth.

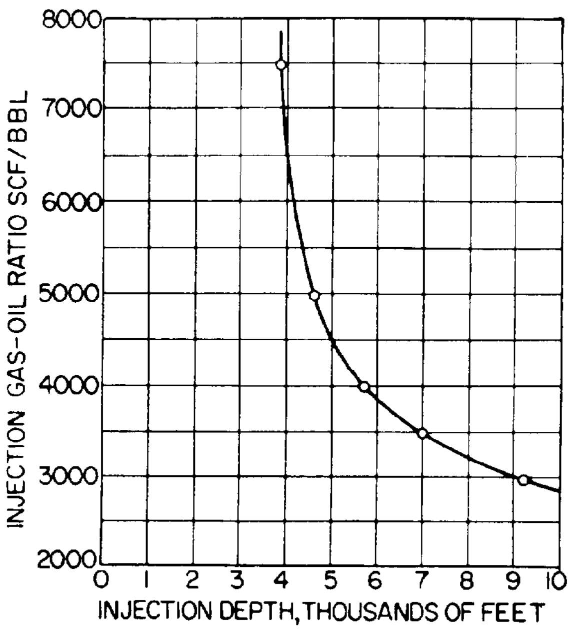

Fig. 34.16—Injection depth vs. injection GOR.

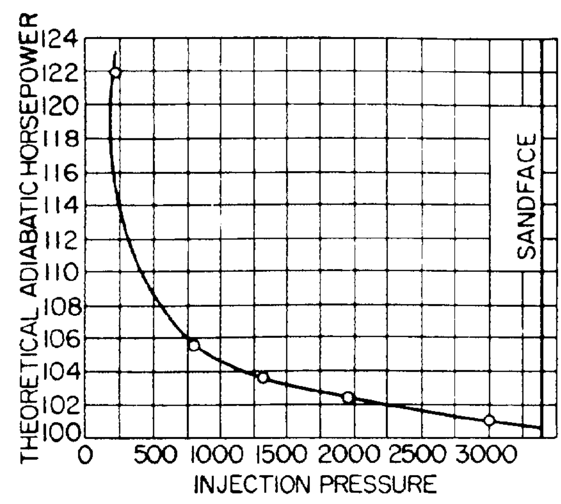

Fig. 34.17—Horsepower vs. injection pressure.

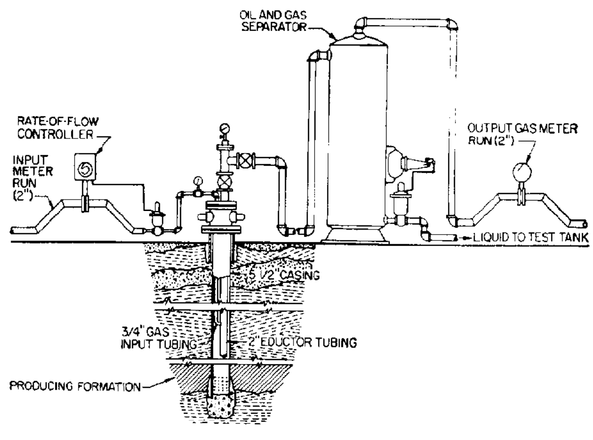

Fig. 34.18—Equipment arrangement.

Fig. 34.17 shows the theoretical adiabatic horsepower required to compress the injected gas from the surface pressure to the injection pressure. For the conditions of this problem, the minimum horsepower is required when the injection point is at the bottom of the well, although, as pointed out in the earlier discussion, it is theoretically possible to obtain minimum horsepower requirements at points other than at the bottom of the hole.

The literature reports an interesting series of well tests in which curves calculated by the procedure described above completely characterize the gas lift performance of the well tested.[43] Fig. 34.18 shows the physical installation of the well tested. Tests were conducted at two points of gas injection, 3,800 and 4,502 ft. Detailed descriptions of the tests are available from Ref. 43.

Figs. 34.19 and 34.20 show a comparison of the observed and calculated pressure traverses above the point of gas injection. The comparison indicates good agreement.

Fig. 34.21 shows a comparison of observed data with curves calculated for average well conditions of total liquid flow vs. rate of gas injection.

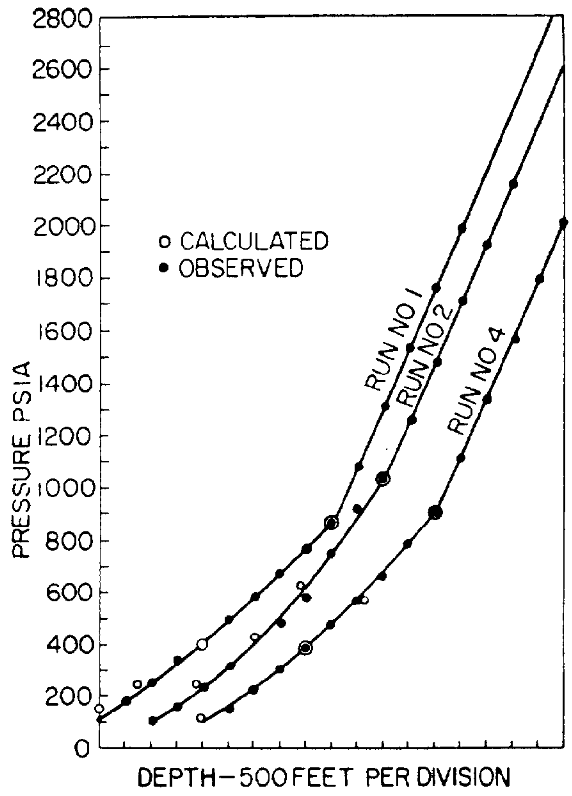

Fig. 34.19—Calculated and field-measured pressure traverses—injection depth is 4,502 ft.

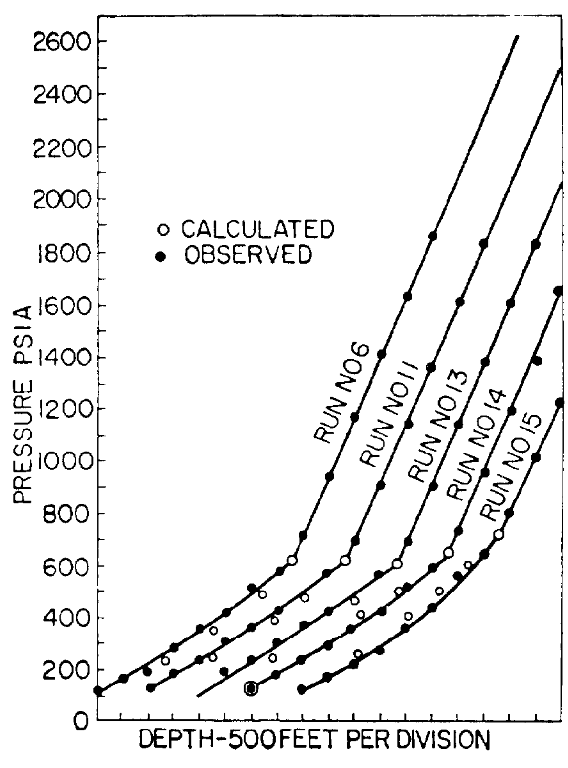

Fig. 34.20—Calculated and field-measured pressure traverses—injection depth is 3,810 ft.

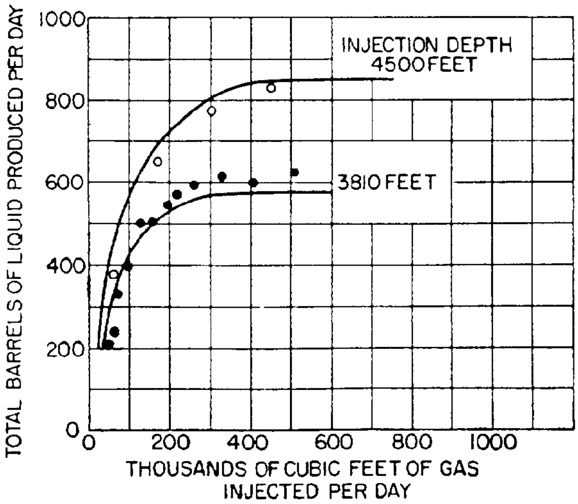

Fig. 34.21—Total liquid flow vs. rate of gas injection.

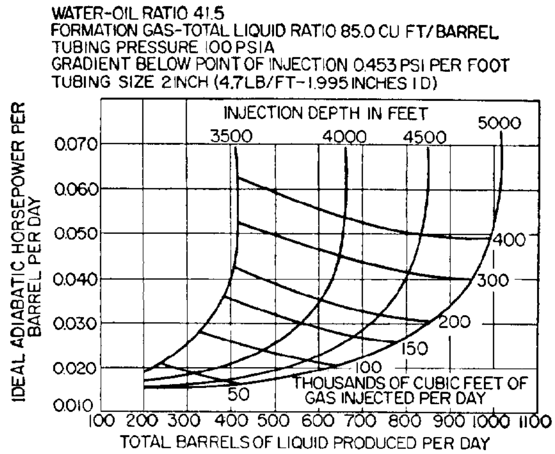

Fig. 34.22—Horsepower requirements vs. total fluid produced.

Fig. 34.22 is an example of a very useful type of plot that can be calculated for the optimum conditions of lift. It is a plot of ideal adiabatic horsepower per barrel per day of total fluid produced vs. total barrels of fluid produced per day under the conditions as indicated. Horsepower as used here is the horsepower required to compress the injected gas between the tubing pressure and injection pressure.

Flow Through Chokes

A wellhead choke or "bean" is used to control the production rate from a well. In the design of tubing and well completions (perforations, etc.), one must ensure that neither the tubing nor perforations control the production from the well. The flow capacity of the tubing and perforations always should be greater than the inflow performance behavior of the reservoir. It is the choke that is designed to control the production rate from a well. Wellhead chokes usually are selected so that fluctuations in the line pressure downstream of the choke have no effect on the well flow rate. To ensure this condition, flow through the choke must be at critical flow conditions; that is, flow through the coke is at the acoustic velocity. For this condition to exist, downstream line pressure must be approximately 0.55 or less of the tubing or upstream pressure. Under these conditions the flow rate is a function of the upstream or tubing pressure only.

For single-phase gas flow through a choke, the following equation is used:

$$q_g=\frac{Cp}{\sqrt{\gamma_g T}}, \quad (121)$$

where

p = upstream pressure, psia,
γ_g = gas gravity,
T = upstream or wellhead temperature, °R,
C = coefficient, and
q_g = flow rate measured at either 14.4 or 14.7 psia and 60°F, 10^3 cu ft/D.

The coefficient, C, will vary depending on the base pressure.

Table 34.7 presents values of C taken from Rawlins and Schellhardt.[44] These values are for a standard pressure of 14.4 psia. Rawlins and Schellhardt did not make corrections for deviation from ideal gas. Correction can be made to Eq. 121 by multiplying the right side of the equation by $\sqrt{1/z}$, where z is the compressibility factor of the gas at the upstream pressure p and temperature T.

In the case of multiphase flow, Gilbert developed the following empirical equation based on data from flowing wells in the Ten Section field of California relating oil flow, GOR, tubing pressure, and choke size.[45]

$$p_{tf}=\frac{435R_{gL}{}^{0.546}q_t}{S^{1.89}}, \quad (122)$$

where

p_{tf} = tubing flowing pressure, psig,
R_{gL} = gas/liquid ratio, 10^3 scf/bbl,
q_t = gross liquid rate (oil and water), B/D, and
S = choke size in 1/64 in.

Gilbert's equation may be written in the form:

$$p_{tf}=Aq_t, \quad (123)$$

TABLE 34.7—COEFFICIENTS FOR CHOKE NIPPLE

Orifice size (in.)		C
1/8	0.125	6.25
3/16	0.188	14.44
1/4	0.250	26.51
5/16	0.313	43.64
3/8	0.375	61.21
7/16	0.438	85.13
1/2	0.500	112.72
5/8	0.625	179.74
3/4	0.750	260.99

where $A=435R_{gL}^{0.546}/S^{1.89}$ and where the tubing pressure is proportional to the production rate. This is true only under conditions of acoustic flow through the choke. At low flow rates, the rate is also a function of the downstream pressure and Eq. 123 no longer holds.

Ros presented a theoretical analysis on the mechanism of simultaneous flow of gas and liquid through a restriction at acoustic velocity.[46,47] The result was a complex equation relating mass flow of gas and liquid, restriction size, and upstream pressure. Ros' equation was checked against oilfield data under critical flow conditions with good results. However, the equation is expressed in a form not really amenable to use by oilfield personnel.

Using Ros' analysis, Poettmann and Beck converted Ros' equation to oilfield units and reduced it to graphical form.[48] The result was Figs. 34.23 through 34.25 for oil gravities of 20, 30, and 40°API. The 20° gravity chart should be used for gravities ranging from 15 to 24°API; similarly, the 30° chart should be used for gravities ranging from 25 to 34°, and the 40° chart for gravities ranging from 35° on up. The charts are not valid if there is appreciable water production with the oil.

The charts can be entered from either the top or bottom scale. When entering from the GOR scale, go first to the tubing pressure curve and then horizontally to the choke size curve and then read the oil flow rate from the top scale. Conversely, when entering the chart at the oil flow rate scale, the reverse order is followed. Reliable estimates of gas rates, oil rates, tubing pressures, and choke sizes can be made by using these charts.

Chokes are subject to sand and gas cutting as well as asphalt and wax deposition, which changes the shape and size of the choke. This, then, could result in considerable error when compared to calculated values of flow for a standard choke size. A small error in choke size caused by a worn choke can effect a considerable error in the predicted oil rate. Thus, a cut choke could result in estimated oil rates considerably lower than measured.

From the inflow performance relationship of a well and by knowing the tubing size in the well, the tubing pressure curve for various flow rates can be calculated. The intersection of the choke performance curve for different choke sizes with the tubing pressure curve then gives one the wellhead pressures and flow rates for any choke size, as illustrated in Fig. 34.26.

Example Problem 10.[48]

1. Determine the flow rate from a well flowing through a $^{6}/_{64}$-in. choke at a flowing tubing pressure of 1,264 psia and a producing GOR of 2,250 cu ft/bbl. Stock-tank gravity is 44.4°. From Fig. 34.25, the solution is 60 B/D oil.

2. For this example, estimate the free gas present in the tubing. The solution gas at a tubing pressure of 1,264 psia from Fig. 34.25 is $R_s=310$ cu ft/bbl. Then, the free gas present is $R-R_s=2{,}250-310$ or 1,940 cu ft/bbl of oil at the wellhead.

3. It is desired to produce a well at 100 B/D oil. The producing GOR is 4,000 cu ft/bbl. At this rate the tubing pressure is 1,800 psia. Estimate choke size.

All three charts show estimated choke size to be $^{8}/_{64}$ in. Gilbert's charts also give $^{8}/_{64}$ in.[45]

A number of other choke design correlations have been suggested. However, Poettmann and Beck's adaption of the Ros equation is recommended when no water is produced with the oil, and Gilbert's equation can be used when water is present.

Liquid Loading in Wells

Liquid loading in wells occurs when the gas phase does not provide sufficient transport energy to lift the liquids out of the well. This type of well does not produce at a flow rate large enough to keep the liquids moving at the same velocity as the gas. The accumulation of liquid will impose an additional backpressure on the formation that can affect the production capacity of the well significantly. Initially, the occurrence of liquid holdup may be reflected in the backpressure data obtained on a well wherein at the lower flow rates its performance, expressed as a backpressure curve, is worse than expected. Eventually, the well is likely to experience "heading" (fluctuating flow rates) followed by "load up" and cease to produce. Methods sometimes used to continue production from "loading" wells are pumping units, plunger lifts, smaller-diameter tubing, soap injection, and flow controllers.

This section is directed mainly toward relating loading to flow conditions within the well. In the simplest context, loading, as reflected on a deterioration of flow performance at lower flow rates on a backpressure curve, is related to the superficial velocity of the gas in the conduit at wellhead conditions. Duggan[49] found that a velocity of 5 ft/sec would keep wells unloaded whereas Lisbon and Henry[50] found that 1,000 ft/min (16.7 ft/sec) could be required.

R.V. Smith[51] reported that experience with low-pressure wells in the West Panhandle and Hugoton fields showed that a velocity of 5 to 10 ft/sec is necessary to remove hydrocarbon liquids consistently and a velocity of 10 to 20 ft/sec is required for water.

Turner *et al.*[52] analyzed the problem of liquid holdup on the basis of two proposed physical models: (1) liquid film movement along the walls of the pipe and (2) liquid droplets entrained in the high-velocity core. They concluded, on the basis of comparisons with field data, that the entrained drop movement was the controlling mechanism for removal of liquids. Their results indicated that in most instances wellhead conditions were controlling and the fluid velocity required to remove liquids could be expressed by the following equation.

$$v_t=\frac{20.4\sigma^{0.25}(\rho_L-\rho_g)^{0.25}}{\rho_g^{\;0.5}}, \quad \text{.............. (124)}$$

where

v_t = terminal velocity of free-falling particle, ft/sec,
σ = interfacial tension, dynes/cm,
ρ_g = gas phase density, lbm/cu ft, and
ρ_L = liquid phase density, lbm/cu ft.

Using simplifying assumptions with respect to gas, condensate, and water properties as given in Table 34.8, Eq. 124 can be expressed for water as

$$v_{gw}=\frac{5.62(67-0.0031p)^{0.25}}{(0.0031p)^{0.5}} \quad \text{.............. (125)}$$

(continued on Page 34-50)

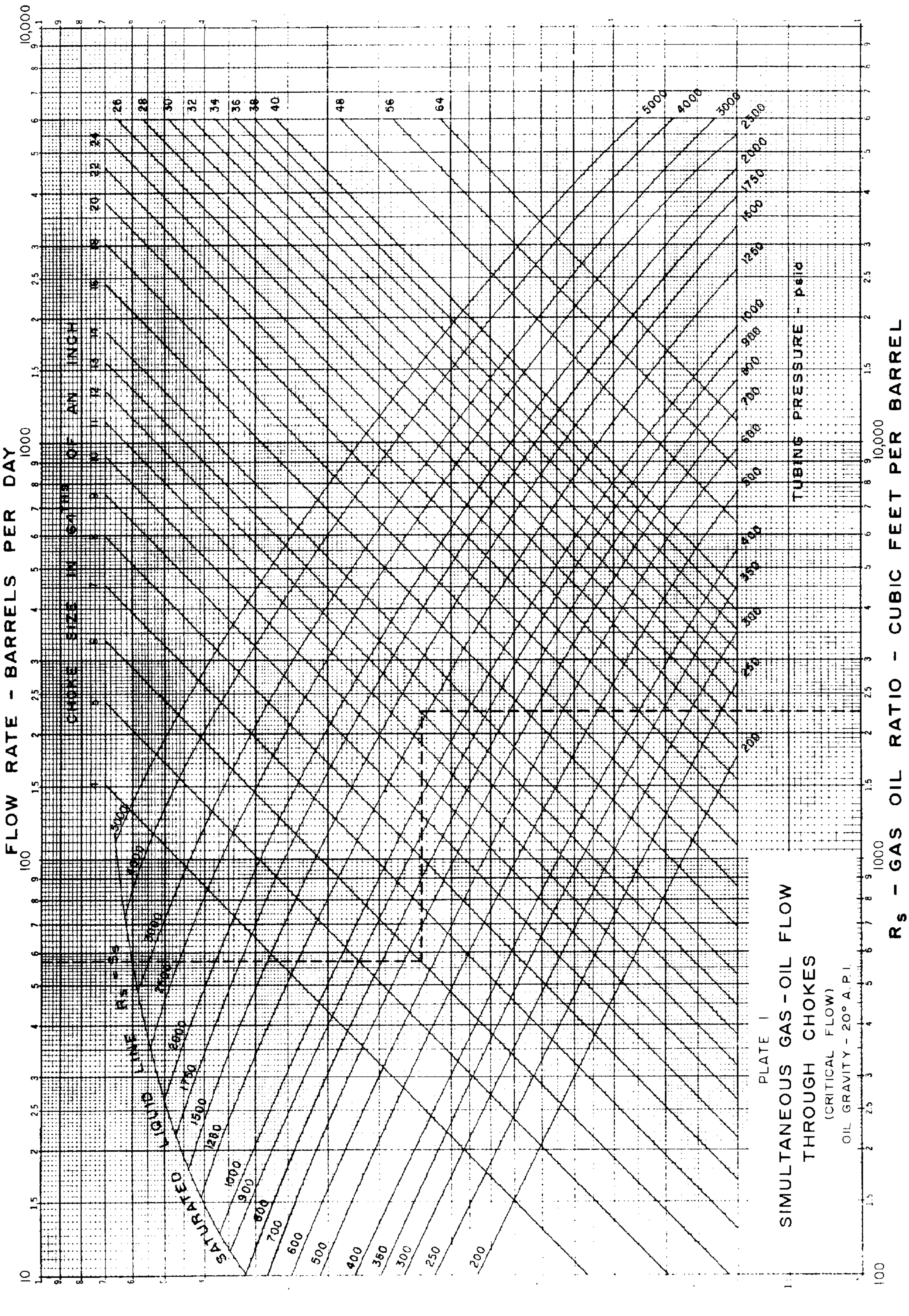

Fig. 34.23—Simultaneous gas/oil flow through chokes.

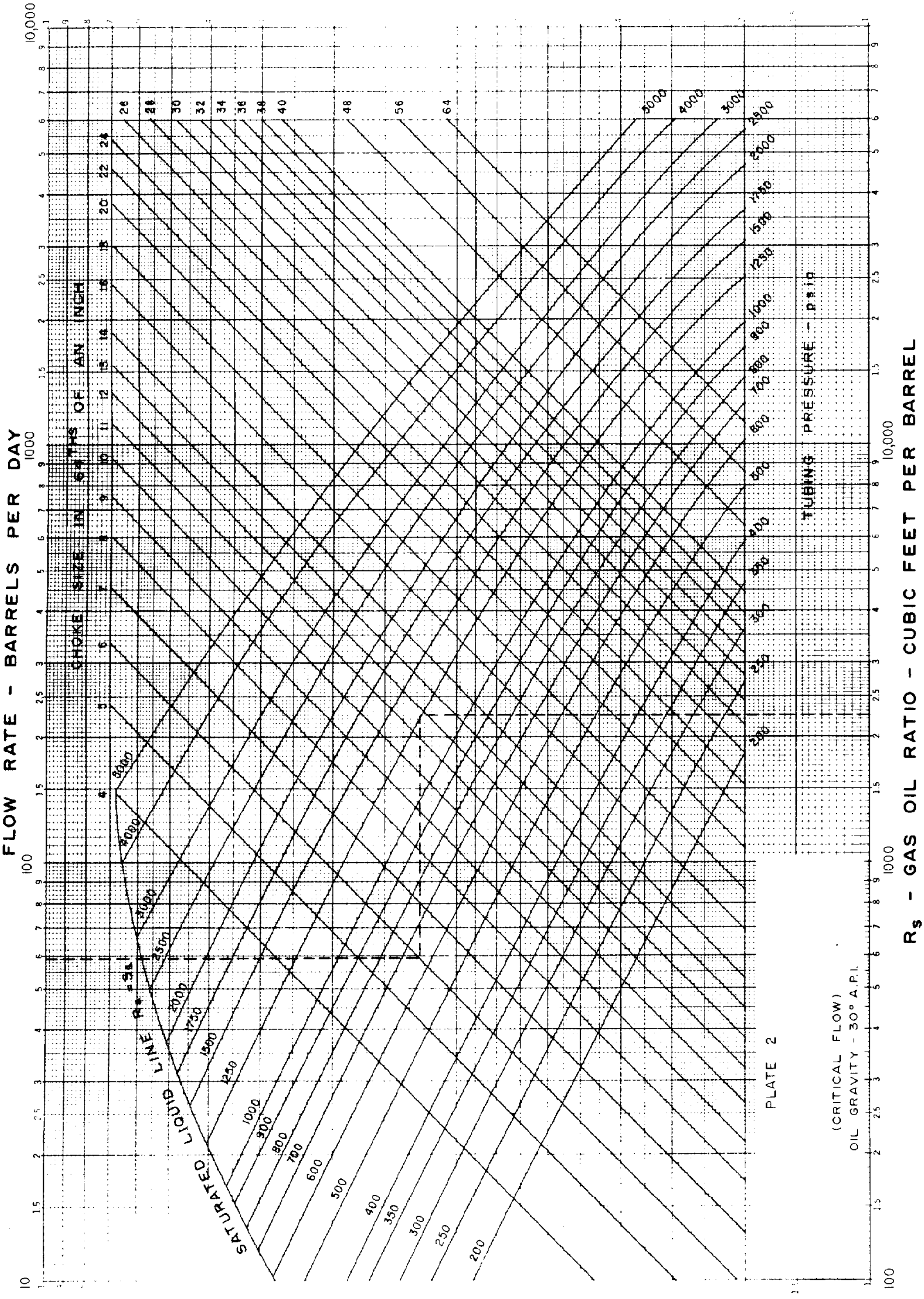

Fig. 34.24—Simultaneous gas/oil flow through chokes.

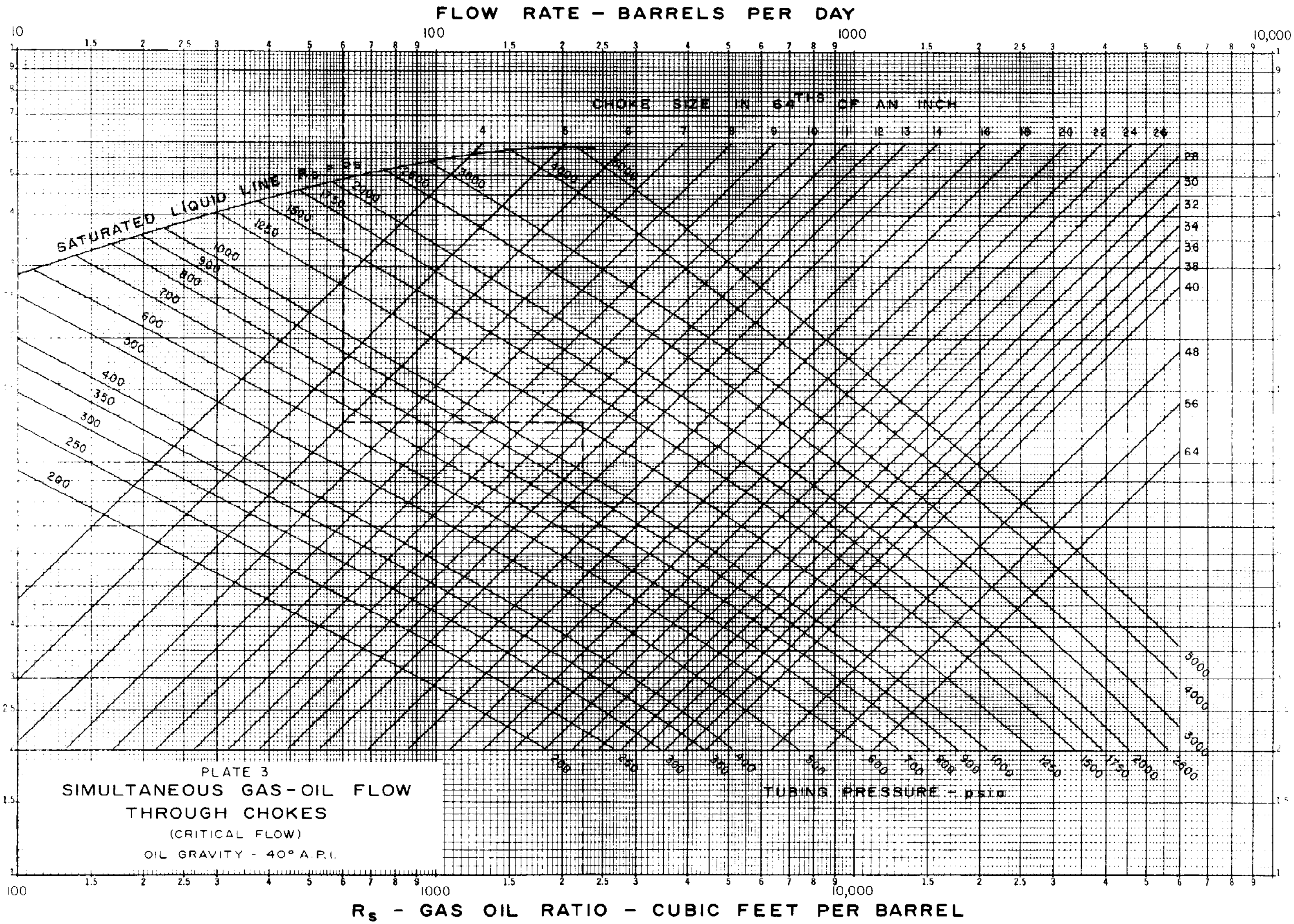

Fig. 34.25—Simultaneous gas/oil flow through chokes.

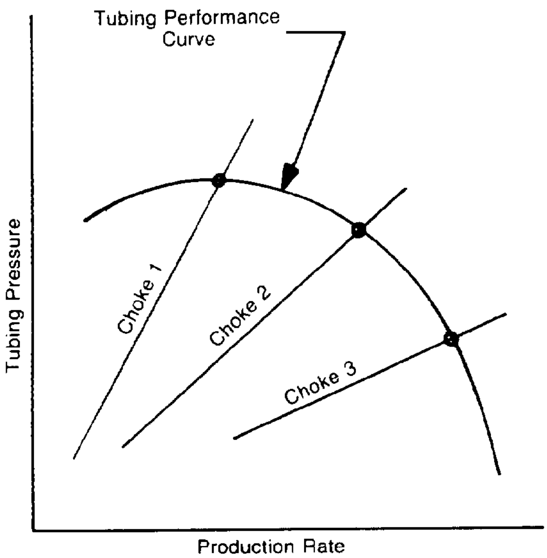

Fig. 34.26—Tubing and choke performance curves.

and for condensate as

$$v_{gc}=\frac{4.02(45-0.0031p)^{0.25}}{(0.0031p)^{0.5}}, \quad \ldots\ldots\ldots\ldots (126)$$

where

v_{gw} = gas velocity for water, ft/sec,
v_{gc} = gas velocity for condensate, ft/sec, and
p = pressure, psi.

Further, a minimum flow rate for a particular set of conditions (pressure and conduit geometry) can be calculated using Eqs. 125 through 127.

$$q_g=\frac{3.06pv_gA}{Tz}, \quad \ldots\ldots\ldots\ldots (127)$$

where

q_g = gas flow rate, 10^6 scf/D,
A = flow area of conduit, sq ft,
T = temperature, °R, and
z = gas deviation factor.

Tek *et al.*[53] introduced a concept called "the lifting potential" to explain loading, unloading, heading, and dying of wells. Further, the concept relates the inflow behavior of the well with the multiphase flow in the well. Accordingly, it appears possible to address engineering considerations directed toward performance analysis or design of well equipment. Calculation procedures described earlier in this chapter with respect to well inflow performance and multiphase flow in the well should be adaptable to use the lifting potential concept.

TABLE 34.8—GAS, CONDENSATE, AND WATER PROPERTIES

	Gas	Condensate	Water
Interfacial tension, dynes/cm	—	20	60
Liquid phase density, lbm/cu ft	—	45	67
Gas gravity	0.6		
Gas temperature, °F	120		

Nomenclature

a,b = constants
A = flow area of conduit
A_c = area under curve

$$B = \frac{667 f_g^{\,2}\bar{T}^2}{d_i^{\,5} p_{pc}^{\,2}} \text{(see Eq. 33)}$$

C_1 = bubble-rise coefficient
C_2 = coefficient, function of liquid velocity
d_{ci} = inside diameter of casing
d_{eq} = diameter of an equivalent circular pipe
d_H = hydraulic pipe diameter
d_{ti} = ID of tubing
d_{to} = OD of tubing
ΔD_i = the *i*th depth increment
D_s = D under static conditions (static equivalent depth for pressures encountered at flowing conditions)
E_ℓ = irreversible energy losses
f = friction factor (Fig. 34.2)
f_f = Fanning friction factor

$$F = F_r q_g = \frac{0.10797 q_g}{d_t^{\,2.612}} \text{(see Eq. 38)}$$

F_{Da} = non-Darcy flow term

$$F_r = \frac{E}{q_g} \text{(see Eq. 38)}$$

F_1 = function of Reynolds number
F_2 = function of Reynolds number and relative roughness
g_c = conversion factor of 32.174

$$I = \frac{p/(Tz)}{F^2+0.001[p/(Tz)]^2} \text{(see Eqs. 40–43)}$$

J^* = stabilized PI at zero drawdown
J' = stabilized PI
J^*_f = stabilized PI at zero drawdown, from future flow data
J^*_p = stabilized PI at zero drawdown, from present flow data
J^*_t = a transient form of the flow coefficient
L = length of the pipe string (subscripts *B*, *M*, and *S* indicate bubble, mist, and slug flow)
L = flow regime boundary, dimensionless
n = exponent, usually determined from multipoint or isochronal backpressure test
N_{Re_b} = bubble Reynolds number
p_b = bubblepoint pressure

p_{bh} = BHP
p_e = reservoir pressure at the external boundary
Δp_i = pressure drop for increment i

$$p_m = p_1 + \frac{p_1 + p_2}{p_2}$$

p_{tf} = tubing flowing pressure
p_{th} = tophole pressure
p_1 = surface pressure
p_2 = bottomhole pressure at depth D
q_{of} = future oil rate
$q_{o(\max)}$ = maximum producing rate at $p_{wf}=0$
Q = heat absorbed by system from surroundings
r_H = hydraulic radius
R_{gL} = gas-liquid ratio
s = skin effect, dimensionless
s = exponent of

$$e = \frac{0.0375\gamma_g L}{\bar{T}\bar{z}} \text{ (see Eq. 44)}$$

S = choke size in 1/64 in.
T_{LM} = log mean temperature
T_1, T_2 = respectively, bottomhole and wellhead temperatures
U = internal energy
v_b = slip or bubble rise velocity
v_{gc} = gas velocity for condensate
v_{gD} = dimensionless gas velocity
v_{gs} = superficial gas velocity
v_{gw} = gas velocity for water
v_{Ls} = superficial liquid velocity
v_t = terminal velocity of free-falling particle
v_t = total fluid velocity (q_t/A)
w_t = total mass flow rate
z = compressibility factor or gas deviation factor
Z = difference in elevation
$(\gamma_g)_{sp}$ = separator gas gravity (air=1)
γ_L = specific gravity of condensate
δ = liquid distribution coefficient
ϵ = absolute roughness
σ = liquid surface tension
τ_f = friction loss gradient

Metric Conversion for Key Equations

Eq. 21

Customary.

$$\int_{0.2}^{(p_{pr})_1} \frac{z}{p_{pr}} dp_{pr} = \frac{L\gamma_g}{53.241\bar{T}} + \int_{0.2}^{(p_{pr})_2} \frac{z}{p_{pr}} dp_{pr}.$$

SI.

$$\int_{0.2}^{(p_{pr})_1} \frac{z}{p_{pr}} dp_{pr} = \frac{L\gamma_g}{29.27\bar{T}} + \int_{0.2}^{(p_{pr})_2} \frac{z}{p_{pr}} dp_{pr},$$

where
p = kPa,
L = m, and
T = °K.

Eq. 28

Customary.

$$p_i = p_2 e^{0.01877\gamma_g L/(\bar{T}\bar{z})}.$$

SI.

$$p_1 = p_2 e^{0.0342\gamma_g L/(\bar{T}\bar{z})},$$

where
p = kPa,
L = m, and
T = °K.

Eq. 35

Customary.

$$\int_{0.2}^{(p_{pr})_1} \frac{\left(\frac{z}{p_{pr}}\right) dp_{pr}}{1+B\left(\frac{z}{p_{pr}}\right)^2} - \int_{0.2}^{(p_{pr})_2} \frac{\left(\frac{z}{p_{pr}}\right) dp_{pr}}{1+B\left(\frac{z}{p_{pr}}\right)^2} = \frac{0.01877\gamma_g L}{\bar{T}}.$$

SI.

$$\int_{0.2}^{(p_{pr})_1} \frac{\left(\frac{z}{p_{pr}}\right) dp_{pr}}{1+B\left(\frac{z}{p_{pr}}\right)^2} - \int_{0.2}^{(p_{pr})_2} \frac{\left(\frac{z}{p_{pr}}\right) dp_{pr}}{1+B\left(\frac{z}{p_{pr}}\right)^2} = \frac{0.0342\gamma_g L}{\bar{T}}.$$

$$B = \frac{1.354 f q_g^2 \bar{T}^2}{d^5 p_{pc}^2},$$

where
q_g = 10^6 m^3/d,
T = °K,
d = m, and
p_{pc} = kPa.

Eq. 36*
Customary.

$$18.75\gamma_g L=\int_{p_2}^{p_1}\frac{[p/(Tz)]dp}{F^2+0.001[p/(Tz)]^2}$$

SI.

$$34.4704\gamma_g L=\int_{p_2}^{p_1}\frac{[p/(Tz)]dp}{F^2+0.001[p/(Tz)]^2}$$

Eq. 37*
Customary.

$$F^2=(2.6665 f_f q_g^2)/d_i^{\,5}$$

SI.

$$F^2=(0.0054150 f_f q_g^{\,2})/d_i^{\,5},$$

where
f_f = Fanning friction factor, dimensionless,**
q_g = 10^6 m^3/d,
T = °K,
p = kPa,
d_i = m, and
L = m.

Eq. 44
Customary.

$$P_{bh}^{\,2}-e^s p_{th}^2=\frac{25 f q_g^{\,2}T^2\bar{z}^{\,2}(e^s-1)}{0.0375 d_i^{\,5}}$$

SI.

$$p_{bh}^{\,2}-e^s p_{th}^{\,2}=\frac{1.354 f q_g^{\,2}\bar{T}^2\bar{z}^{\,2}(e^s-1)}{d_i^{\,5}},$$

where
p = kPa,
q_g = 10^6 m^3/d,
f = from Fig. 34.2,
$\bar{T}$ = °K,
d = m,
$s=\dfrac{0.0683\gamma_g L}{\bar{T}\bar{z}}$, and
L = m.

*In using SI units, Table 34.4 and Eqs. 38 and 39 are not applicable.
**f_f is the Fanning friction factor equal to $f_f=f/4$, where f is the Moody friction factor from Fig. 34.2.

Eq. 56
Customary.

$$p_2=p_1+\frac{D\rho}{144}.$$

SI.

$$p_2=p_1+9.8\times10^{-3}D\rho,$$

where
p = kPa,
D = m, and
ρ = kg/m^3.

Eq. 65
Customary.

$$J^*=\frac{7.08kh}{\left[\ln\left(\dfrac{r_e}{r_w}\right)-\dfrac{3}{4}+s\right]}\cdot\frac{k_{ro}}{(\mu_o B_o)_{p_R}}.$$

SI.

$$J^*=\frac{0.0005427kh}{\left(\ln\dfrac{r_e}{r_w}-\dfrac{3}{4}+s\right)}\cdot\frac{k_{ro}}{(\mu_o B_o)_{p_R}},$$

where
J^* = m^3/d-kPa,
h = m, and
μ_o = Pa·s.

Eq. 66
Customary.

$$J^*_{(t)}=\frac{7.08kh}{\left(\ln\sqrt{\dfrac{14.23kt}{\phi\mu c_t r_w^{\,2}}}+s\right)}\left[\left(\frac{k_{ro}}{\mu_o B_o}\right)_{p_R}\right].$$

SI.

$$J^*_{(t)}=\frac{0.0005427kh}{\left(\ln\sqrt{\dfrac{0.009115kt}{\phi\mu c_t r_w^{\,2}}}+s\right)}\left[\left(\frac{k_{ro}}{\mu_o B_o}\right)_{p_R}\right],$$

where
h = m,
t = d,
μ = Pa·s,
c_t = 1/kPa, and
r_w = m.

Eq. 87

Customary.

$$-\mathrm{d}p=\frac{\tau_f \mathrm{d}D}{144}+\frac{g\rho}{144g_c}\mathrm{d}D+\frac{gv}{144g_c}\mathrm{d}v.$$

SI.

$$-\mathrm{d}p=\tau_f \mathrm{d}D+\frac{1000g\rho}{g_c}\mathrm{d}D+\frac{1000\rho v}{g_c}\mathrm{d}v,$$

where
- p = kPa,
- τ_f = kPa/m,
- D = m,
- ρ = g/cm^3,
- g = 9.80 m/s^2,
- g_c = 1000 kg/m·kPa·s^2, and
- v = m/s.

Eq. 89

Customary.

$$\Delta p_i=\frac{1}{144}\,\frac{\rho+\tau_f}{1-\dfrac{w_t q_g}{4637A^2p}}\,\Delta D_i.$$

SI.

$$\Delta p_i=\frac{9.806\rho+\tau_f}{1-\dfrac{w_t q_g}{1000A^2p}}\,\Delta D_i,$$

where
- w_t = kg/s,
- q_g = m^3/s, and
- A = m^2.

Eq. 90

Customary.

$$v_{gD}=\frac{q_g}{A}\left(\frac{\rho_L}{g\sigma}\right)^{0.25}$$

SI.

$$v_{gD}=\frac{q_g}{A}\left(\frac{10^6\rho_L}{g\sigma}\right)^{0.25},$$

where
- q_g = m^3/s,
- A = m^2,
- ρ_L = g/cm^3,
- g = 9.8 m/s^2, and
- σ = g/s^2.

Eq. 91

Customary.

$$L_B=1.071-\frac{0.2218v_t^{\,2}}{d_H}.$$

SI.

$$L_B=1.071-\frac{0.7277v_t^{\,2}}{d_H},$$

where
- v_t = m/s and
- d_H = m.

Eq. 98

Customary.

$$N_{\mathrm{Re}}=\frac{1{,}488\rho_L d_H v_L}{\mu_L}.$$

SI.

$$N_{\mathrm{Re}}=\frac{1000\rho_L d_H v_L}{\mu_L},$$

where
- ρ_L = g/m^3,
- d_H = m,
- v_L = m/s, and
- μ_L = Pa·s.

Eq. 101

Customary.

$$N_{\mathrm{Re}}=\frac{1{,}488\rho_L d_H v_b}{\mu_L}$$

SI.

$$N_{\mathrm{Re}}=\frac{1000\rho_L d_H v_b}{\mu_L}.$$

Eq. 102

Customary.

$$N_{\mathrm{Re}}=\frac{1{,}488\rho_L d_H v_t}{\mu_L}.$$

SI.

$$N_{\mathrm{Re}}=\frac{1000\rho_L d_H v_t}{\mu_L}.$$

Eq. 117
Customary.

$$N_{Re}=\frac{1{,}488\rho_g d_H v_{gs}}{\mu_g}.$$

SI.

$$N_{Re}=\frac{1000\ \rho_g d_H v_{gs}}{\mu_g}.$$

Eq. 118
Customary.

$$N=4.52(10^{-7})\left(\frac{v_{gs}\mu_L}{\sigma}\right)^2\left(\frac{\rho_g}{\rho_L}\right).$$

SI.

$$N=10^6\left(\frac{v_{gs}\mu_L}{\sigma}\right)^2\left(\frac{\rho_g}{\rho_L}\right),$$

where
v_{gs} = m/s,
μ_L = Pa·s, and
σ = g/s^2.

Eq. 119
Customary.

$$\frac{\epsilon}{d_H}=\frac{34\sigma}{\rho_g v_{gs}^2 d_H}.$$

SI.

$$\frac{\epsilon}{d_H}=\frac{1.115(10^{-4})\sigma}{\rho_g v_{gs}^2 d_H},$$

where
σ = g/s^2,
v_{gs} = m/s, and
ρ_g = g/cm^3.

Eq. 120
Customary.

$$\frac{\epsilon}{d_H}=\frac{174.8\sigma(N)^{0.302}}{\rho_g v_{gs}^2 d_H}.$$

SI.

$$\frac{\epsilon}{d_H}=\frac{5.735(10^{-4})\sigma(N)^{0.302}}{\rho_g v_{gs}^2 d_H},$$

where
σ = g/s^2,
v_{gs} = m/s, and
ρ_g = g/cm^3.

Eq. 121
Customary.

$$q_g=\frac{Cp}{\sqrt{\gamma_g T}}.$$

SI.

$$q_g=\frac{3.0169Cp}{\sqrt{\gamma_g T}},$$

where
q_g = m^3/d,
T = °K, and
p = kPa.

Eq. 122
Customary.

$$p_{tf}=\frac{435R_{gL}^{0.546}q_t}{S^{1.89}}.$$

SI.

$$p_{tf}=\frac{2.50R_{gL}^{0.546}q_t}{S^{1.89}},$$

where
p_{tf} = kPa,
R_{gL} = m^3/m^3,
q_t = m^3/d, and
S = cm.

Eq. 125
Customary.

$$v_{gw}=\frac{5.62(67-0.0031p)^{0.25}}{(0.0031p)^{0.5}}.$$

SI.

$$v_{gw}=\frac{1.713(67-0.00045p)^{0.25}}{(0.00045p)^{0.5}}.$$

Eq. 126
Customary.

$$v_{gc}=\frac{4.02(45-0.0031p)^{0.25}}{(0.0031p)^{0.5}}.$$

SI.

$$v_{gc}=\frac{1.225(45-0.00045p)^{0.25}}{(0.00045p)^{0.5}},$$

where

p = kPa and
v_g = m/s.

Eq. 127

Customary.

$$q_g=\frac{3.06pv_gA}{Tz}.$$

SI.

$$q_g=\frac{0.24628^*pv_gA}{Tz},$$

where

p = kPa,
v_g = m/s,
A = m^2,
T = °K, and
q_g = 10^6 m^3/d.

*Based on standard conditions of 520°R and 14.7 psia.

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